香 港 大 學
THE UNIVERSITY OF HONG KONG

A General Introduction

Prof. Li Jiayang

DASE 7113 Optimization Methods for Intelligent Systems

About Me

Education



2015 - 2019

Tsinghua University

B.S. in Mathematics

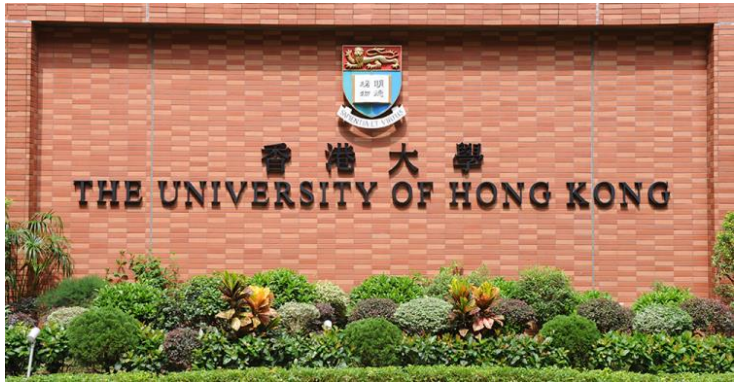


2019 - 2024

Northwestern University

Ph.D. in Civil and Environmental Engineering

Appointment



2024 - now

The University of Hong Kong

Assistant Professor of Data and Systems Engineering

About the Course

Venue	K.K. Leung Building 202
Time	7:00 PM ~ 9:50 PM, Monday
Instructor	Prof. Jiayang LI (jiayangl@hku.hk)
Office Hour	By appointment

Project Description

1. This is a group project (3-4 students per group).

2. Each group needs to write a report about 4000 words and make a presentation.

3. In the report, you need to identify a practical optimization problem, which may comes from any domain (*literally! but not from the lecturers, of course*). The report must includes two parts: (1) an introduction of how the problem is motivated and formulated (and other details you found interesting); (2) the application of 1-2 algorithms for solving one or more numerical example.

About Optimization

What is optimization?

A quest for the best solution to achieve certain objective given certain constraints.

1. Decision variable:	What you are optimizing?	→	x
2. Objective function:	What you are optimizing for?	→	$f(x)$
3. Constraints:	Rules to follow.	→	$g_i(x) \geq 0, i = 1, \ldots, m$ $h_i(x) = 0, i = 1, \ldots, p$

About Optimization

What is optimization?

A quest for the best solution to achieve certain objective given certain constraints.

1. Decision variable:

2. Objective function:

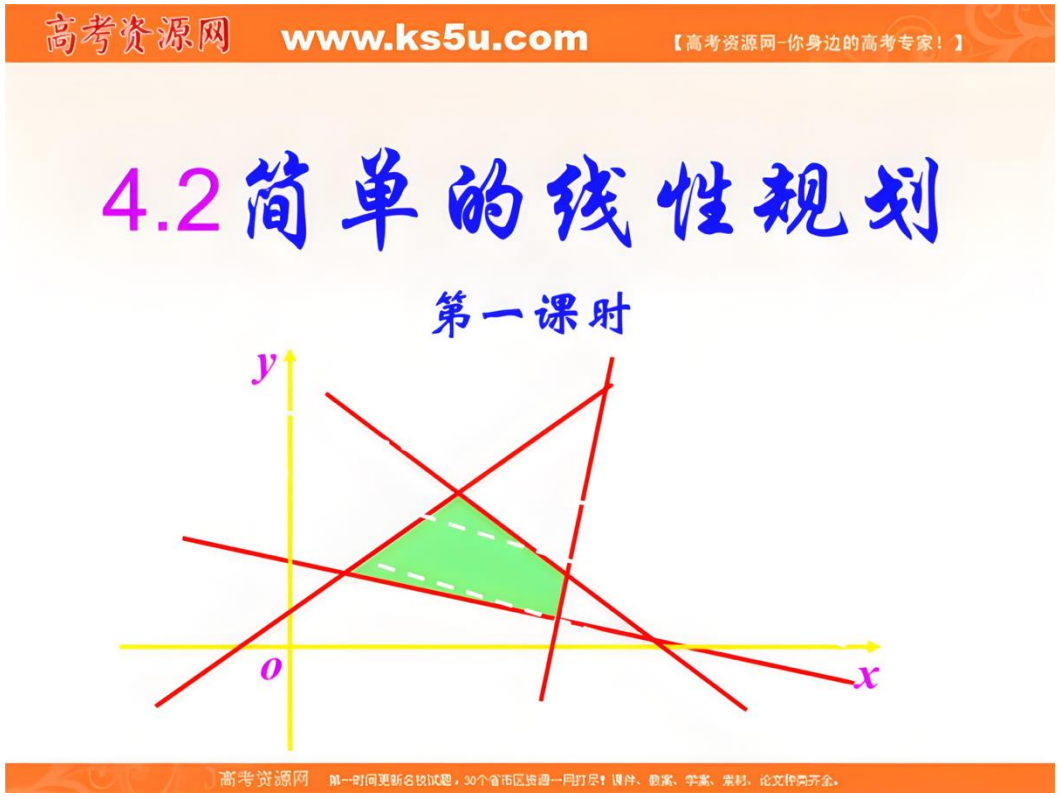
3. Constraints:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \ i = 1, \ldots, m \\ & h_i(x) = 0, \ i = 1, \ldots, p \end{aligned}$$

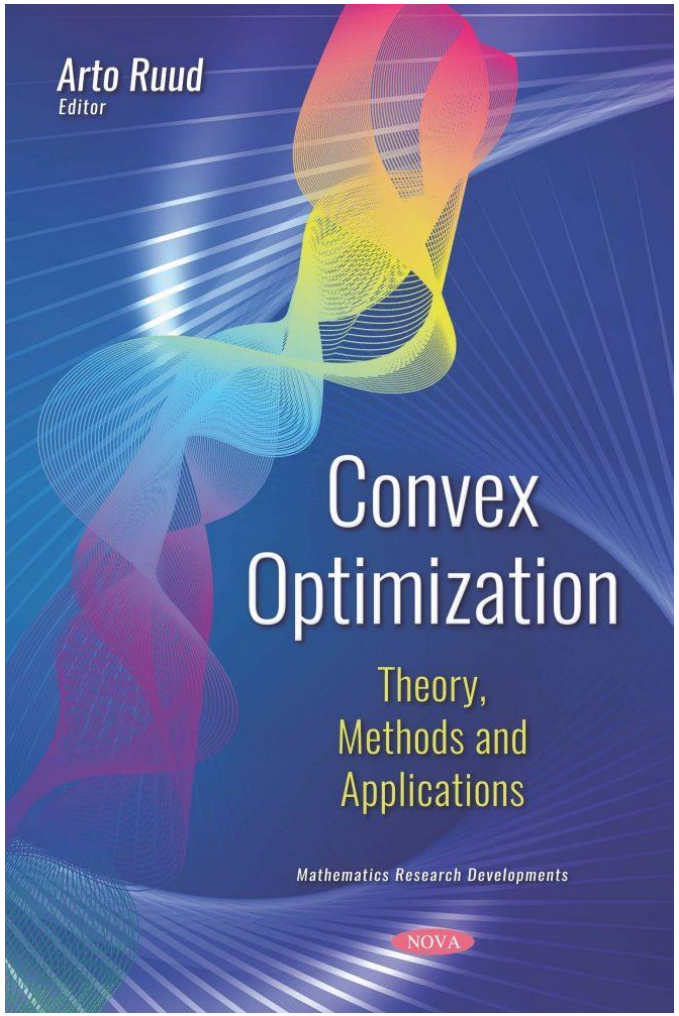
a "mathematical program" ?

About Optimization

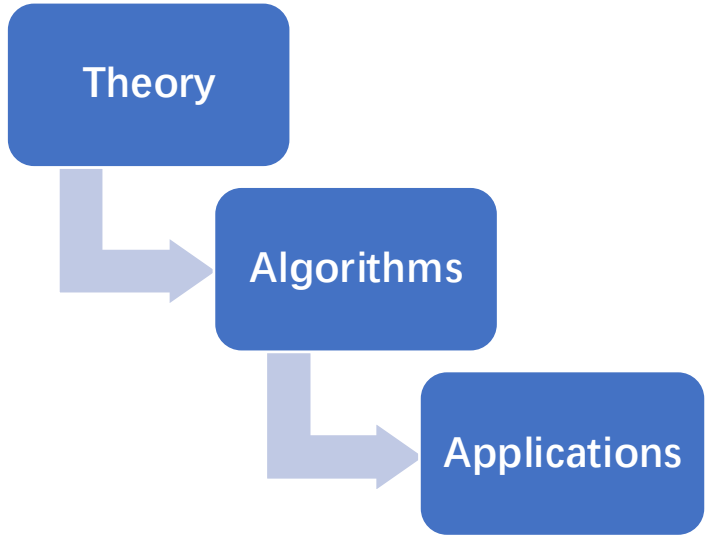
Your first lesson in optimization? That was in high school.



Course Structure



Most optimization course follows a theory-algorithm-application structure.



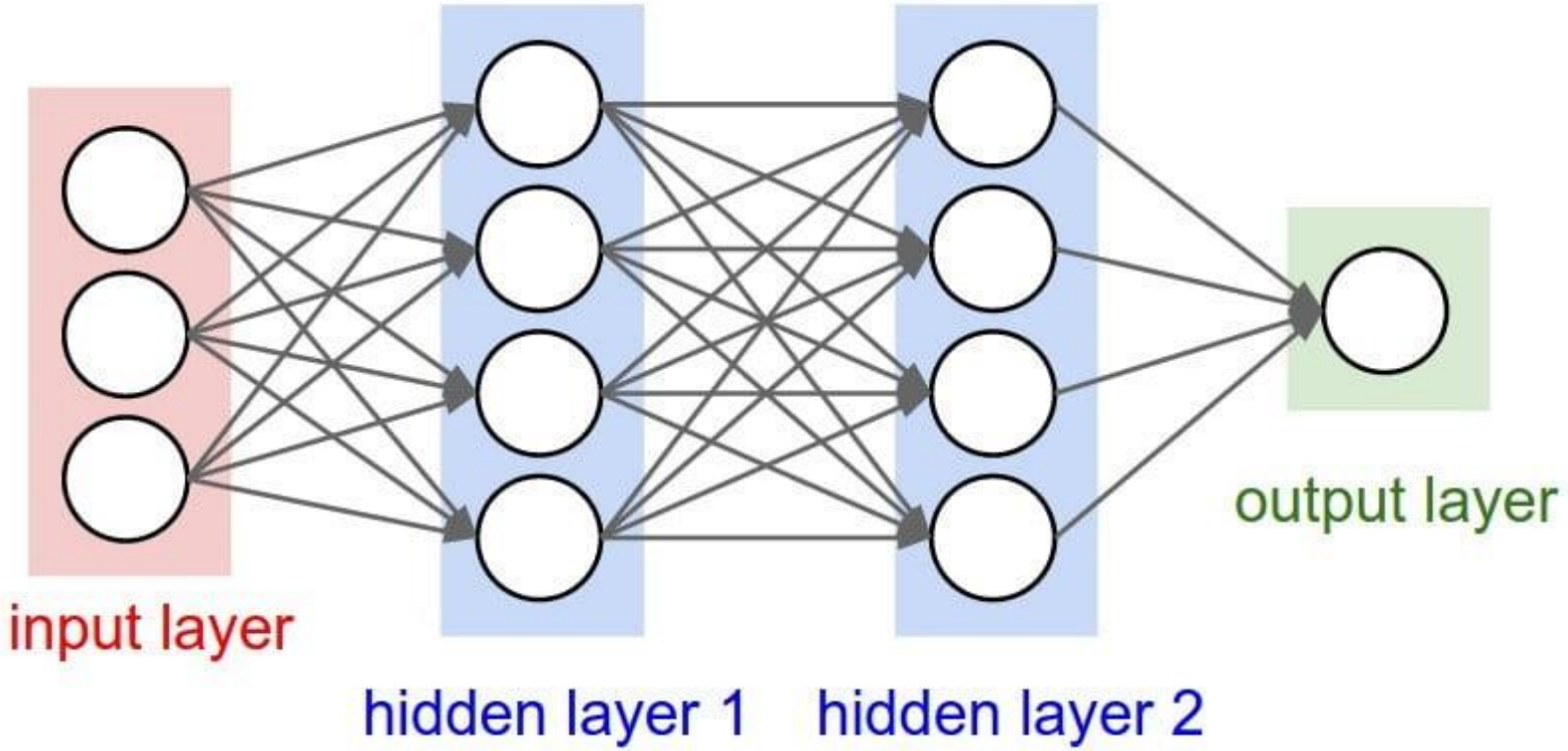
But we will begin with "Applications"

Optimization Is Everywhere

1. Machine Learning and Data Science
2. Supply Chain Management
3. Transportation and Logistics
4. Manufacturing and Production Planning
5. Finance and Investment Management
6. Inventory Management
7. Telecommunications
8. Robotics and Control Systems
9. Healthcare and Medical Decision Making
10. Energy Systems and Power Generation
11. Environmental Management and Sustainability
12. Marketing and Advertisement Strategy
13. Resource Allocation and Scheduling
14. Structural Engineering and Design
15. Computational Biology and Bioinformatics
16. Economics and Game Theory
17. Project Management
18. Dynamic Pricing and Revenue Management
19. Risk Management and Actuarial Science
20. ……

Optimization Is Everywhere

Machine Learning and Data Science



Optimization Is Everywhere

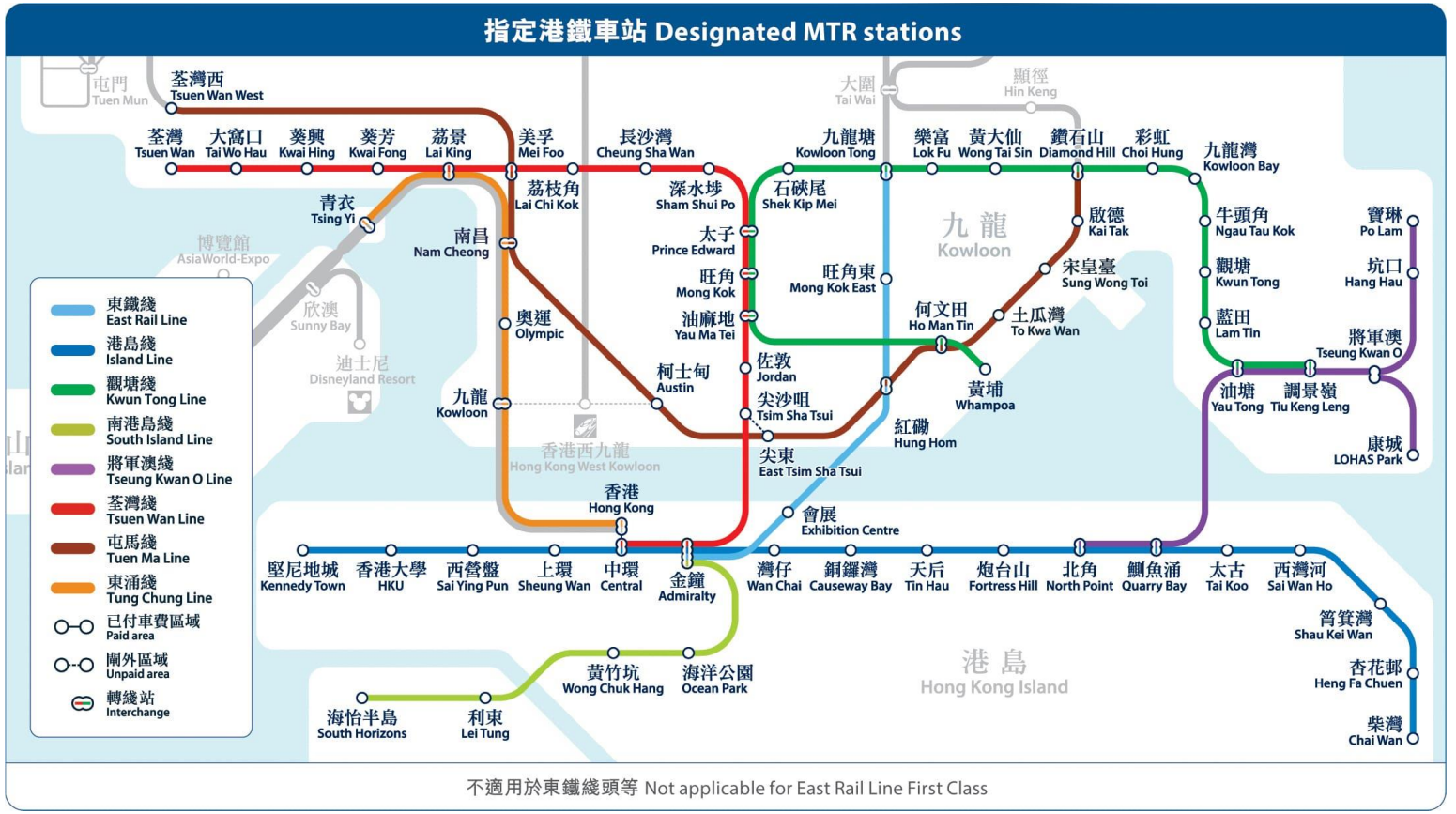
Transportation and Logistics





Optimization Is Everywhere

Transportation and Logistics



Optimization Is Everywhere

Inventory Management



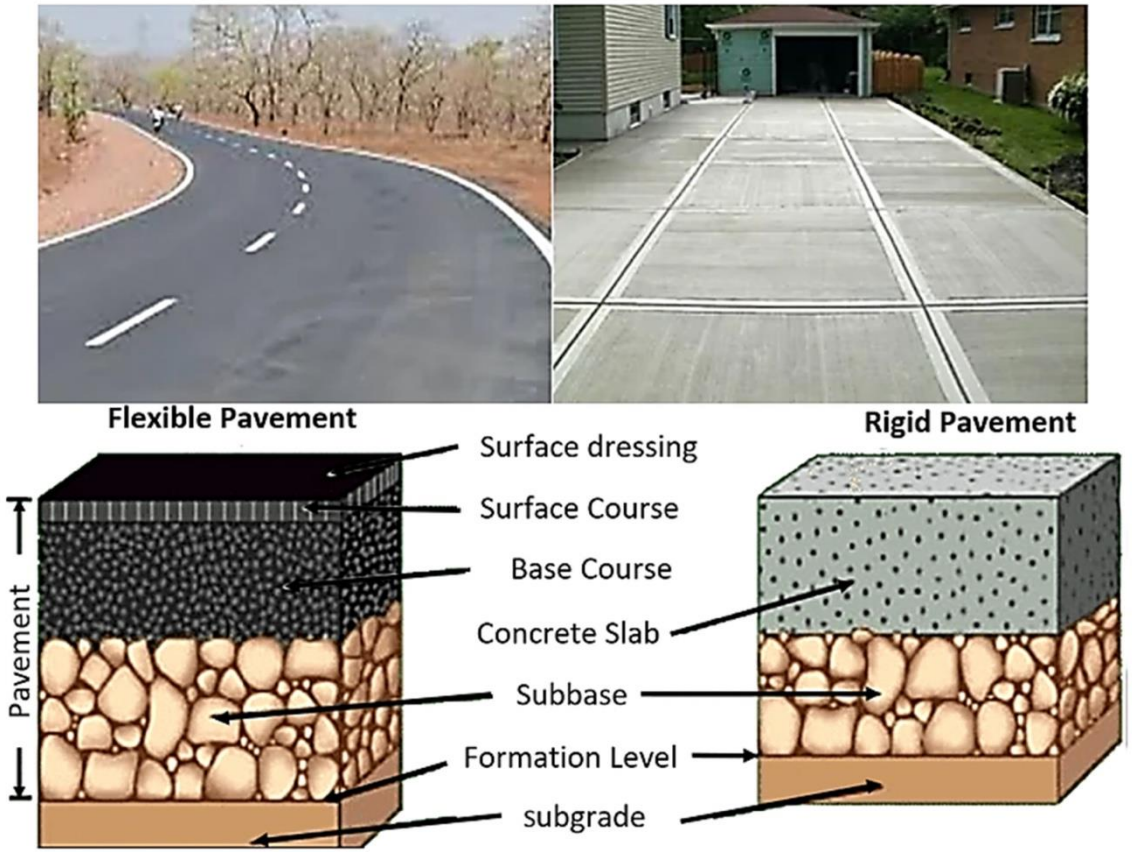
Optimization Is Everywhere

Energy Systems and Power Generation



Optimization Is Everywhere

Structural Engineering and Design



Optimization Is Everywhere

Healthcare and Medical Decision Process



SURGERY SCHEDULE DATE:

Time	OR	Patient	Pat. Room	Procedure	Surgeon	Nurse/Team	Aneth.	An. Type

Notes:

Optimization Is Everywhere

Revenue Management





My Research

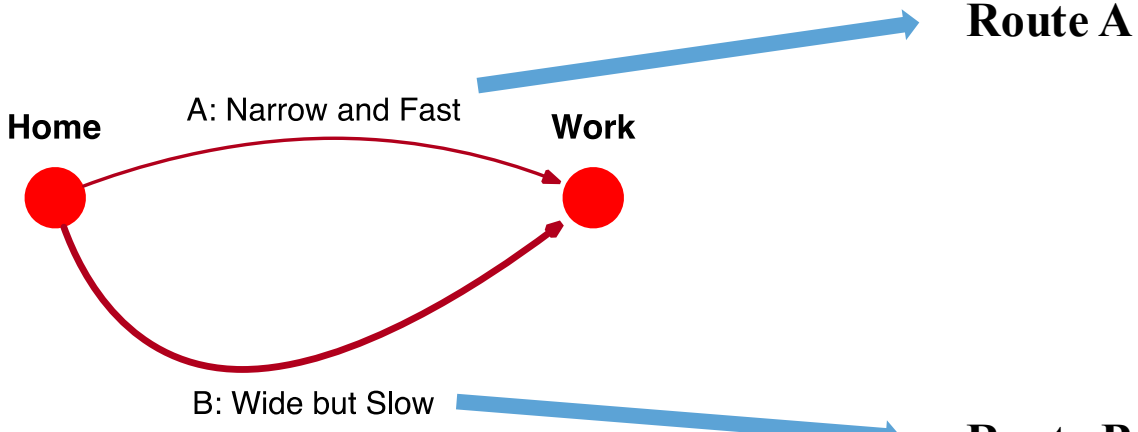
My Research: How to optimize the performance of transportation systems?





The Pigou-Knight Model

Consider a small town with ***n*** **residents** who drive from "Home" to "Work" every morning.



Route A

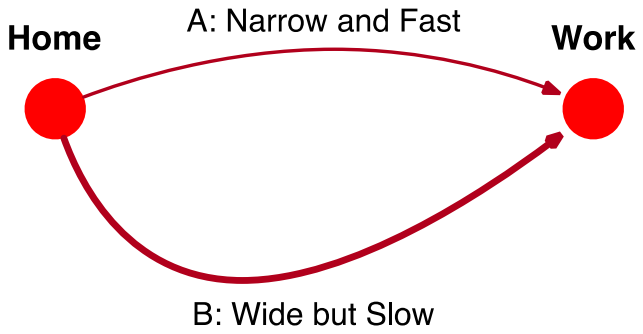


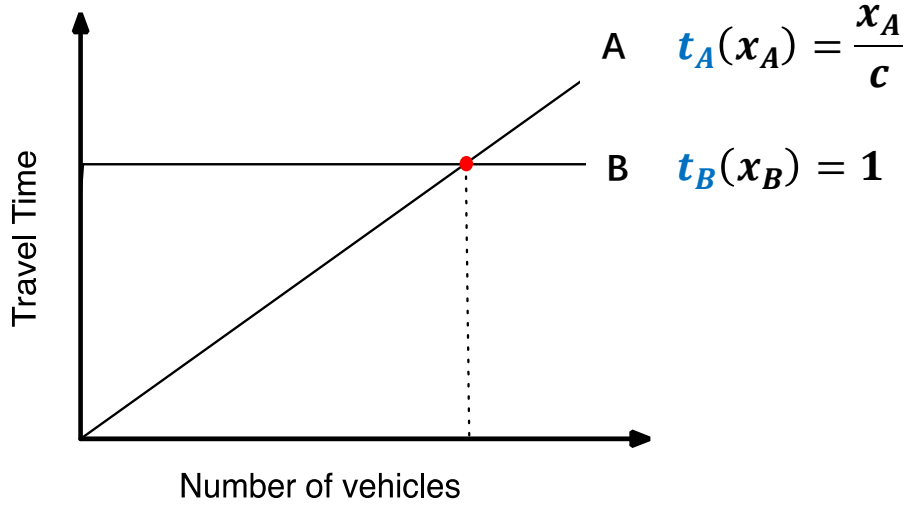
Route B



The Pigou-Knight Model

Consider a small town with n residents who drive from "Home" to "Work" every morning.





x_A: The number of travelers choosing Route A

x_B: The number of travelers choosing Route B

The Pigou-Knight Model

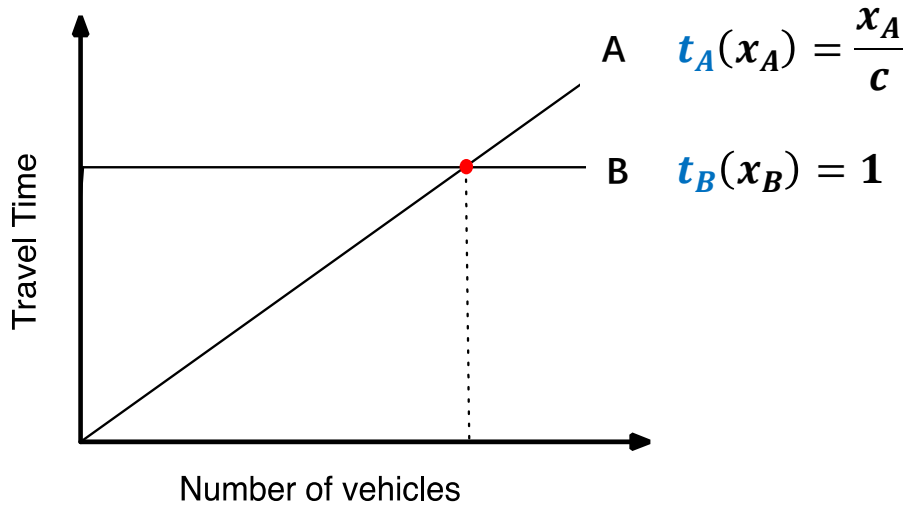
Consider a small town with n residents who drive from "Home" to "Work" every morning.

Question: If $n = 0.5c$, how would the traffic be split?

All choose A ➡ $x_A = 0.5c, x_B = 0$

⬇

$$t_A(x_A) = 0.5 < 1 = t_B(x_B)$$



x_A: The number of travelers choosing Route A

x_B: The number of travelers choosing Route B

The Pigou-Knight Model

Consider a small town with n residents who drive from "Home" to "Work" every morning.

Question: If $n = c$, how would the traffic be split?

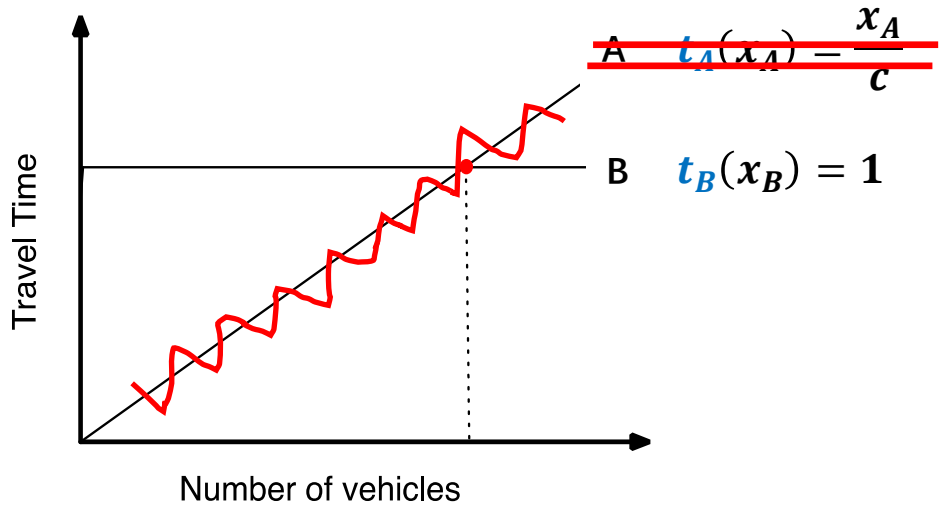
All choose A ➡ $x_A = c, x_B = 0$

⬇

$t_A(x_A) = 1 = 1 = t_B(x_B)$

⬇

The total travel time would be c



x_A: The number of travelers choosing Route A

x_B: The number of travelers choosing Route B

The Pigou-Knight Model

Consider a small town with n residents who drive from "Home" to "Work" every morning.

Question: If $n = c$, how would the traffic be split?

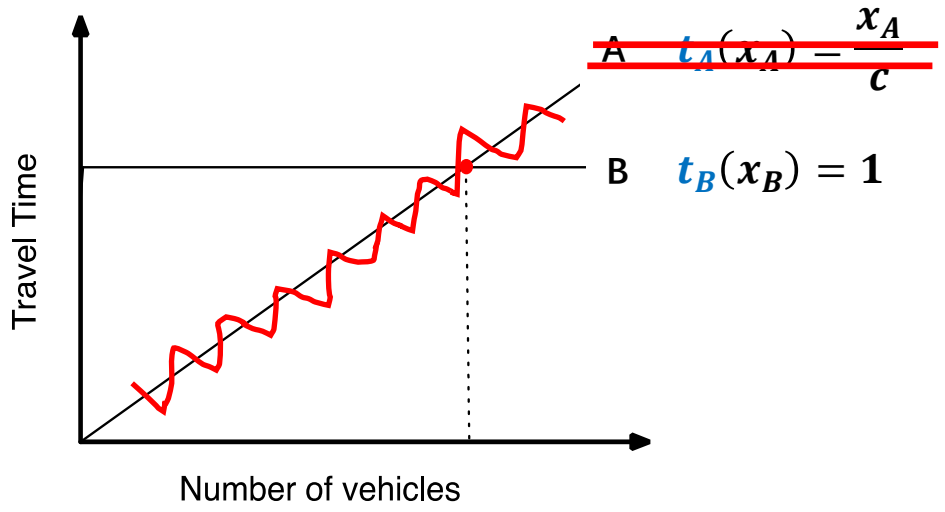
All choose B ⇒ $x_B = n$

⇓

$t_B(x_B) = 1$

⇓

The total travel time would still be c!

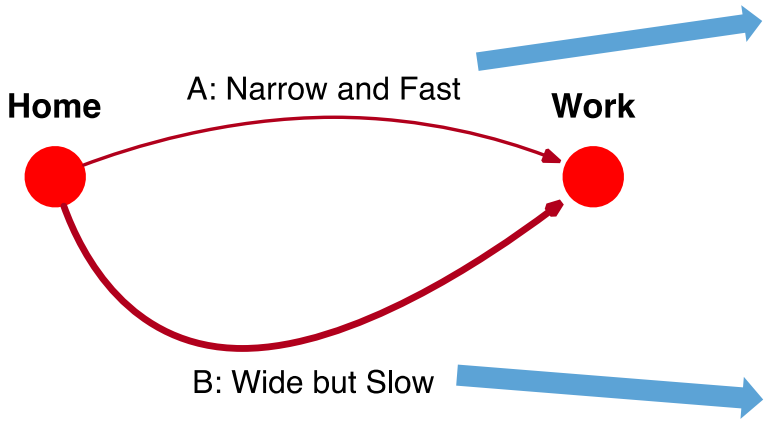


x_A: The number of travelers choosing Route A

x_B: The number of travelers choosing Route B

The Pigou-Knight Model

Consider a small town with ***n*** **residents** who drive from "Home" to "Work" every morning.



Route A



It is used

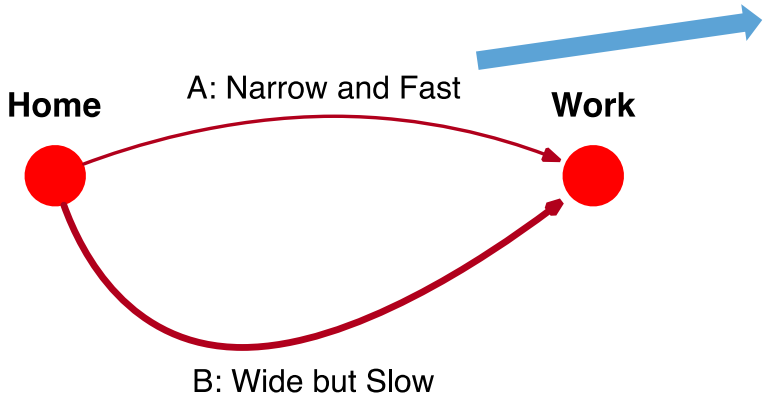
But it is useless !

Route B

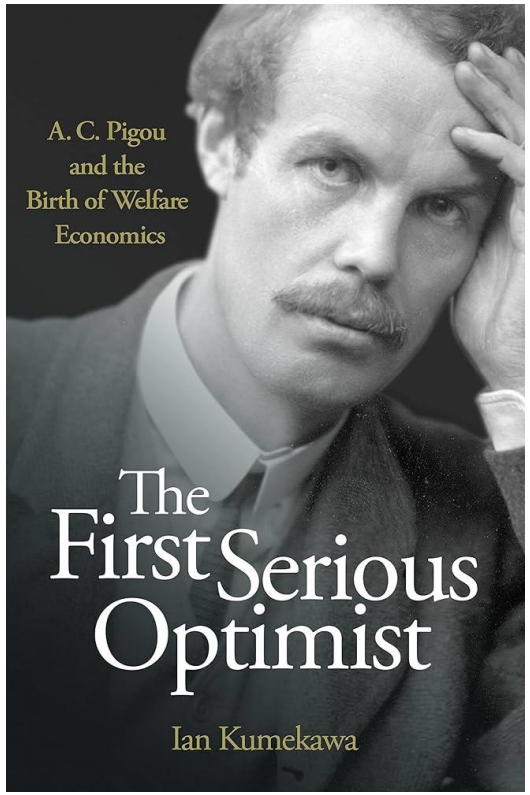


The Pigou-Knight Model

Consider a small town with n residents who drive from "Home" to "Work" every morning.



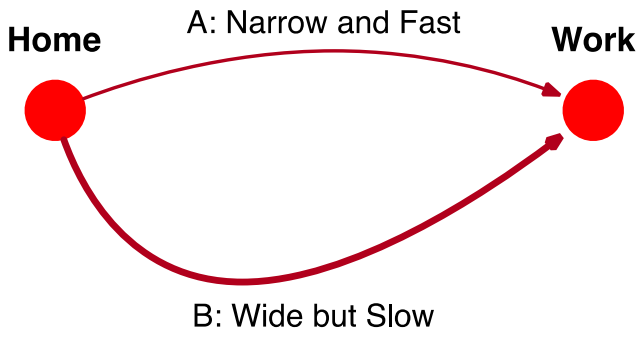
We can charge a toll π on Route A



The Pigou-Knight Model

Consider a small town with $n = c$ residents who drive from "Home" to "Work" every morning.





Question: What is the optimal π that leads to the least congested outcome?

$$\text{minimize} \quad \frac{x_A}{c} \cdot x_A + (c - x_A)$$

$$\pi \geq 0$$

$$\text{subject to} \quad \frac{x_A}{c} + \pi = 1$$

Route A's cost

Route B's cost

The Pigou-Knight Model

Consider a small town with $n = c$ residents who drive from "Home" to "Work" every morning.

Question: What is the optimal π that leads to the least congested outcome?

Optimal toll $\pi^* = \frac{1}{2}$

Total travel time $\frac{3}{4}c < c$



minimize $\frac{x_A}{c} \cdot x_A + (c - x_A)$

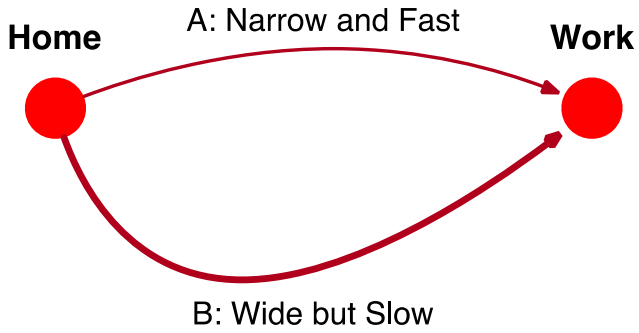
$\pi \geq 0$

subject to $\frac{x_A}{c} + \pi = 1$

Route A's cost

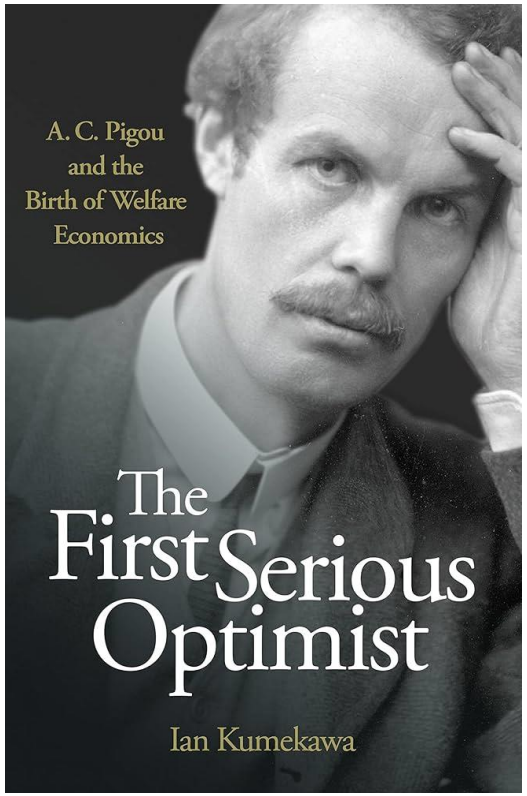
Route B's cost

The Pigou-Knight Model



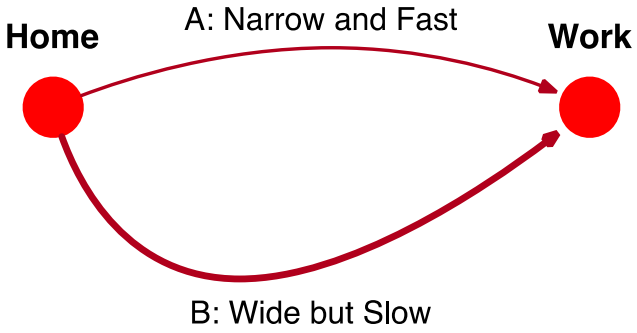
Implication: Government can charge **tolls**

to correct the inefficient use of transportation facilities

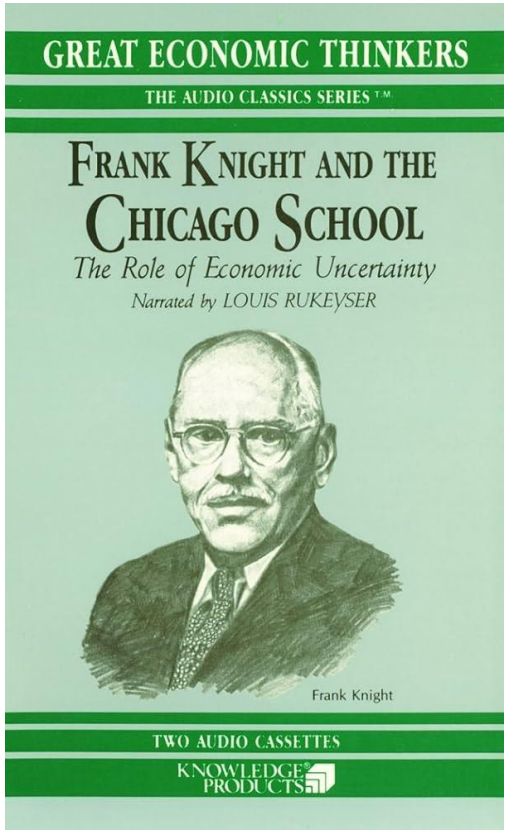


The Pigou-Knight Model

Knight's argument: **If the road is privately owned,**
the government then needs to do nothing!



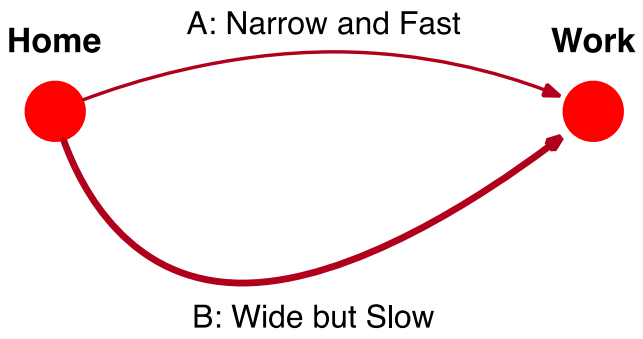




The Pigou-Knight Model

Consider a small town with $n = c$ residents who drive from "Home" to "Work" every morning.





Question: What is the optimal π that leads to the most profitable outcome?

$$\text{maximize}_{\pi \geq 0} \quad \pi \cdot x_A$$

$$\text{subject to} \quad \frac{x_A}{c} + \pi = 1$$

Route A's cost Route B's cost

The Pigou-Knight Model

Consider a small town with $n = c$ residents who drive from "Home" to "Work" every morning.

Question: What is the optimal π that leads to the most profitable outcome?

Identical to Pigou's toll

Optimal toll $\pi^* = \frac{1}{2}$

Total travel time $\frac{3}{4}c < c$

maximize $\pi \cdot x_A$

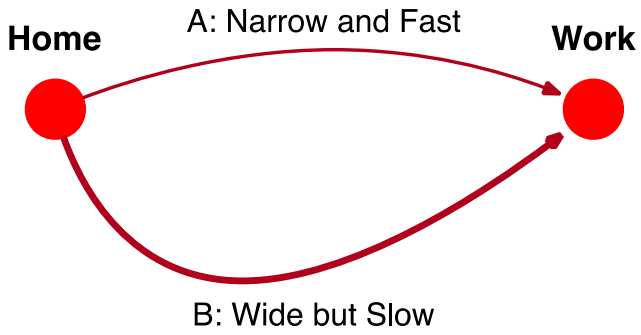
$\pi \geq 0$

subject to $\frac{x_A}{c} + \pi = 1$

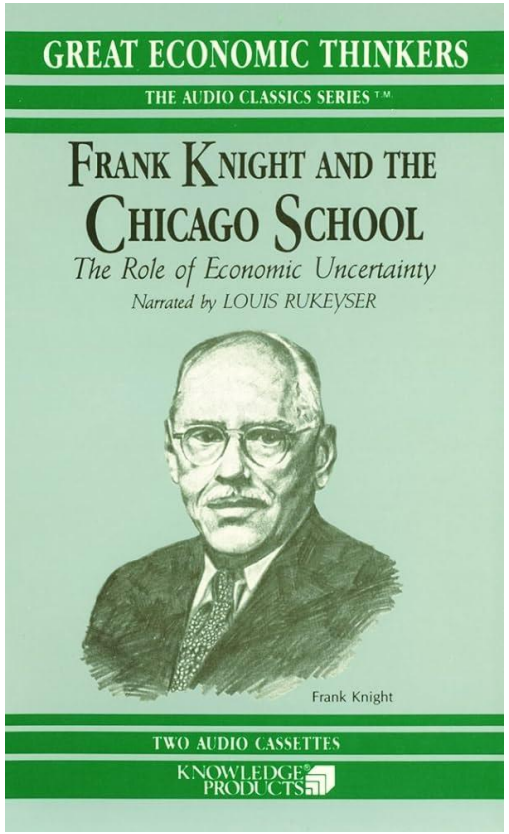
Route A's cost

Route B's cost

The Pigou-Knight Model

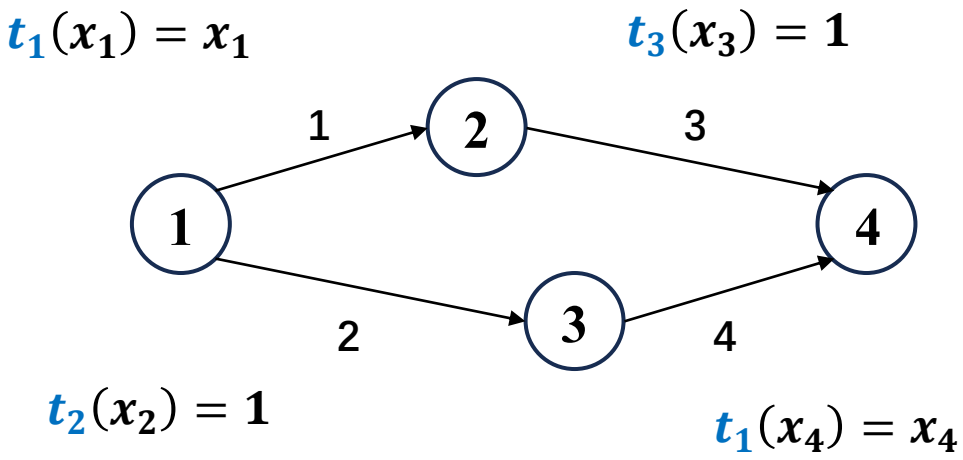


Implication: The public ownership of roads
results in a "market failure" (or government failure)



The Braess Paradox

Consider a small town with 1 unit of residents traveling from Node 1 to Node 4 everyday



$1 \to 2 \to 4$	$1 \to 3 \to 4$
$f_1 = 0.5$	$f_2 = 0.5$

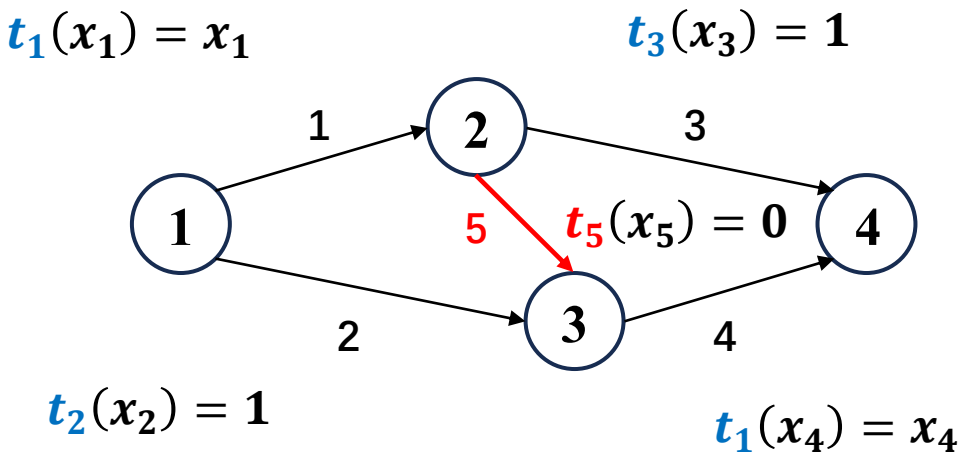
$$x_1 = x_2 = x_3 = x_4 = 0.5$$

Total travel time: 1.5

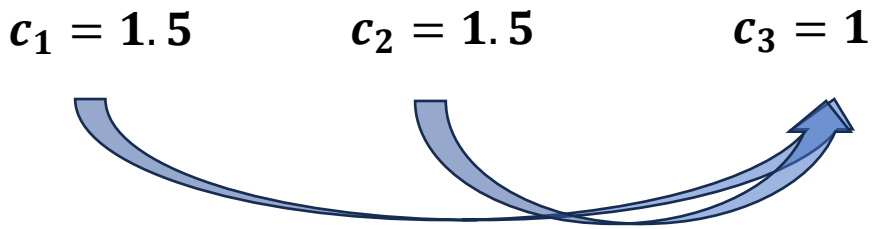
How will travelers be distributed between the two routes ($1 \to 2 \to 4$ and $1 \to 3 \to 4$)?

The Braess Paradox

Consider a small town with 1 unit of residents traveling from Node 1 to Node 4 everyday



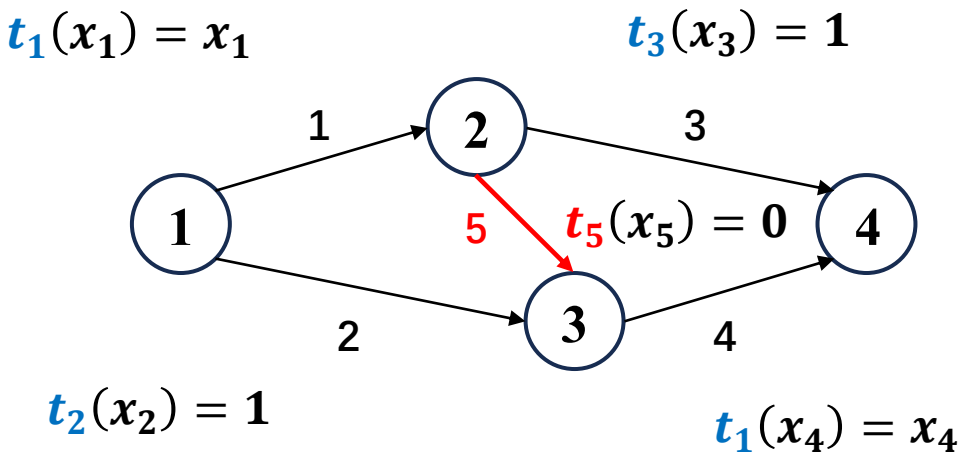
$1 \to 2 \to 4$	$1 \to 3 \to 4$	$1 \to 2 \to 3 \to 4$
$f_1 = 0.5$	$f_2 = 0.5$	$f_3 = 0$



How will travelers will be distributed between the three routes (the original two and $1 \to 2 \to 3 \to 4$)?

The Braess Paradox

Consider a small town with 1 unit of residents traveling from Node 1 to Node 4 everyday



$1 \to 2 \to 4$	$1 \to 3 \to 4$	$1 \to 2 \to 3 \to 4$
$f_1 = 0$	$f_2 = 0$	$f_3 = 1$

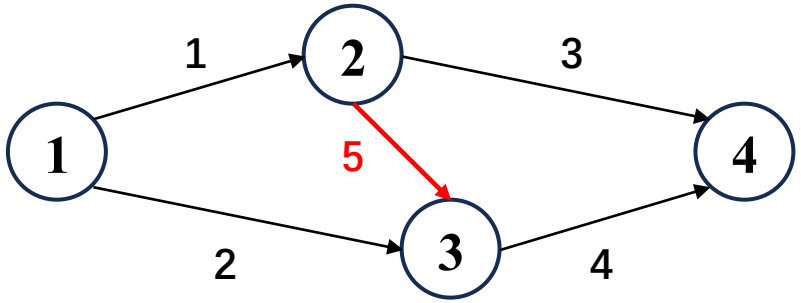
$c_1 = 2$ $c_2 = 2$ $c_3 = 2$

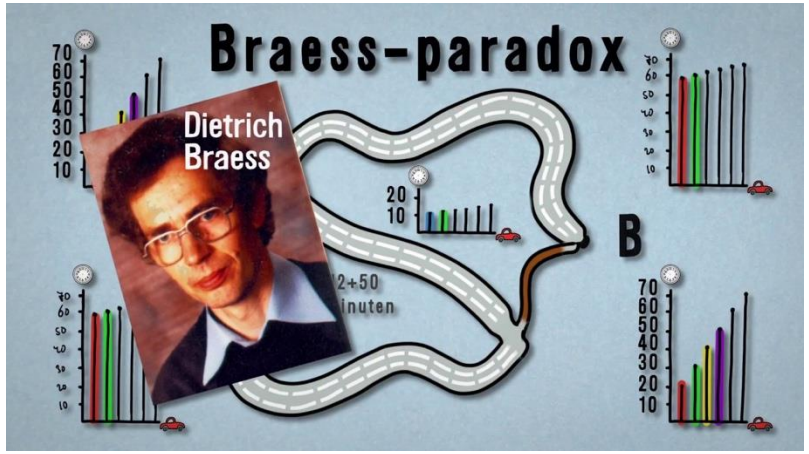
Total travel time: 2 > 1.5

How will travelers will be distributed between the three routes (the original two and $1 \to 2 \to 3 \to 4$)?

The Braess Paradox

Building **more roads** can lead to **more severe congestions**!





Implication: A careful and systematic analysis of highway investment

Network Design Problems

a traffic manager

design road capacities

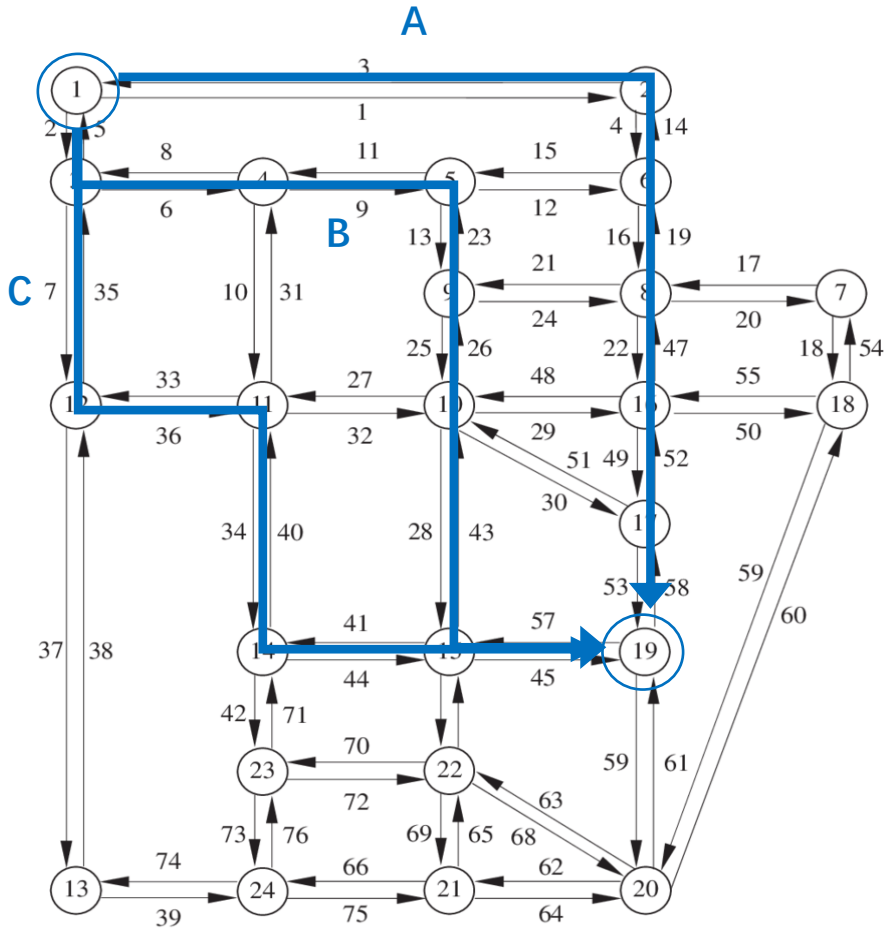


The capacity expansion problem

design congestion tolls



The congestion pricing problem



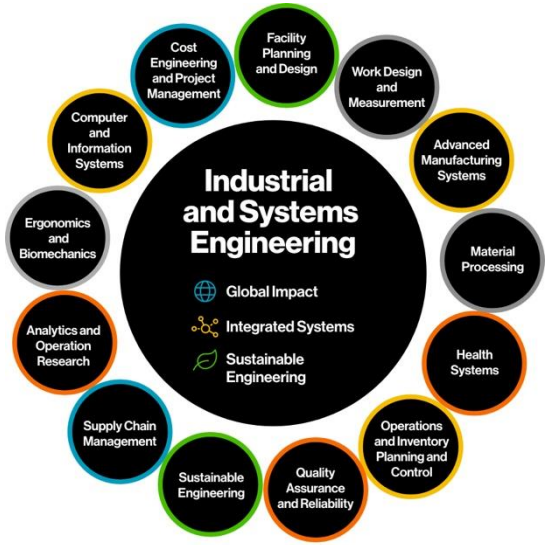
Topics to be Explored

How to formulate optimization problems

Optimization problems in data science

Optimization problems in system engineering





How to solve optimization problems

Unconstrained convex programming algorithms

Constrained convex programming algorithms

Integer programming algorithms

Metaheuristic algorithms