

香 港 大 學
THE UNIVERSITY OF HONG KONG

Optimization Problems in System Engineering

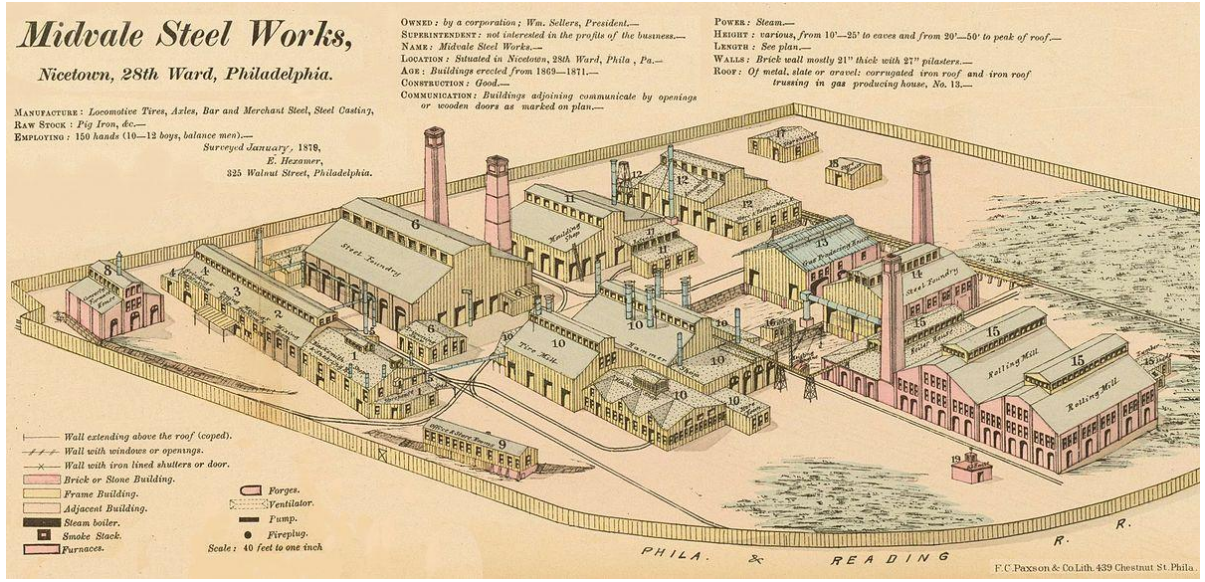
Prof. Li Jiayang

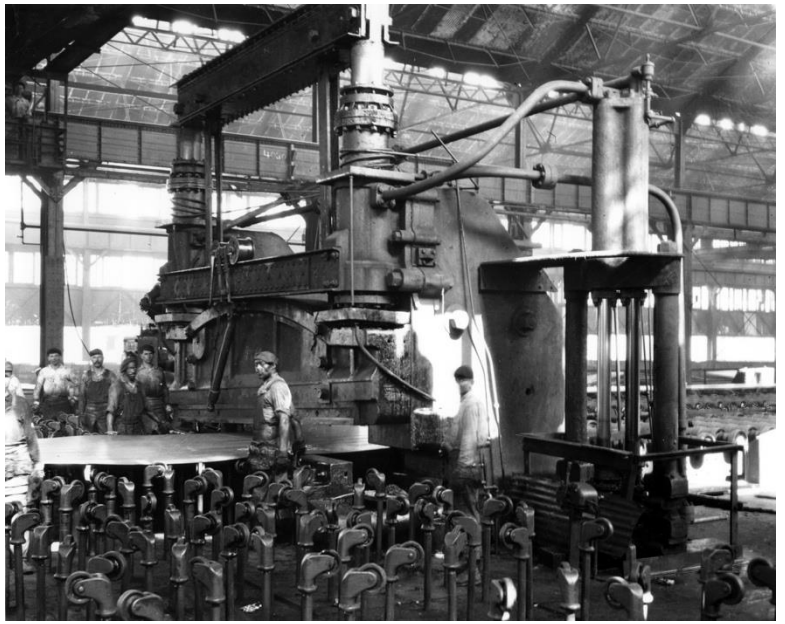
IMSE7113 Optimization Methods for Intelligent Systems

Industrial Engineering

Definition

Industrial engineering is an engineering profession that focuses on **optimizing** products, services, processes, and operations across various systems.





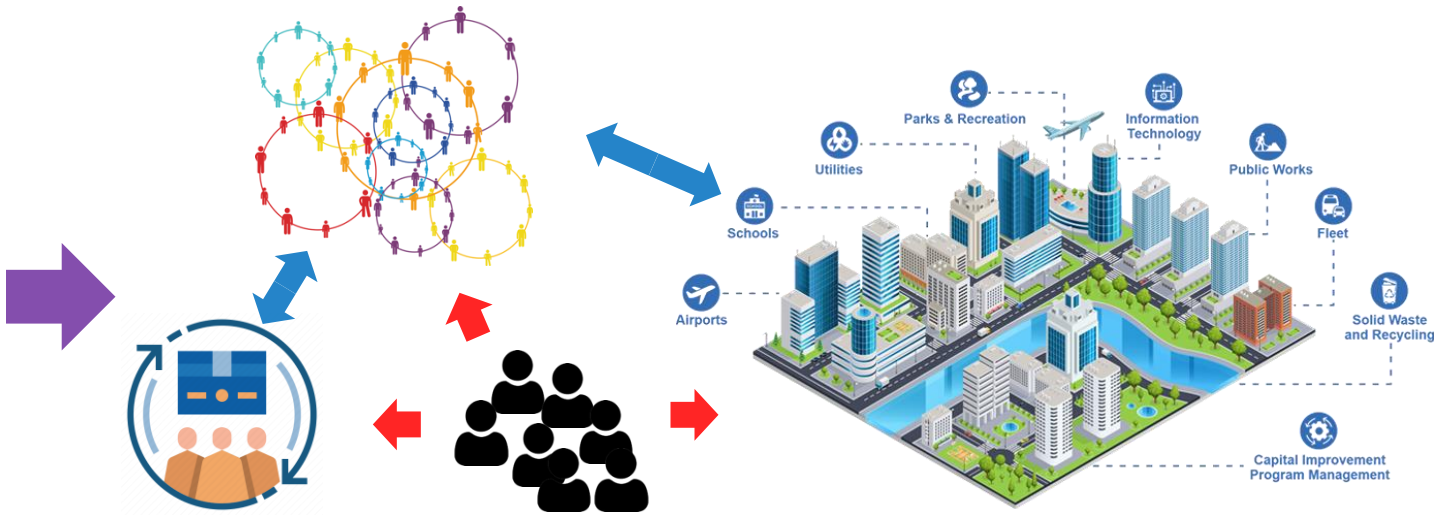
Origin: How should workshops be managed so as the production efficiency can be optimized

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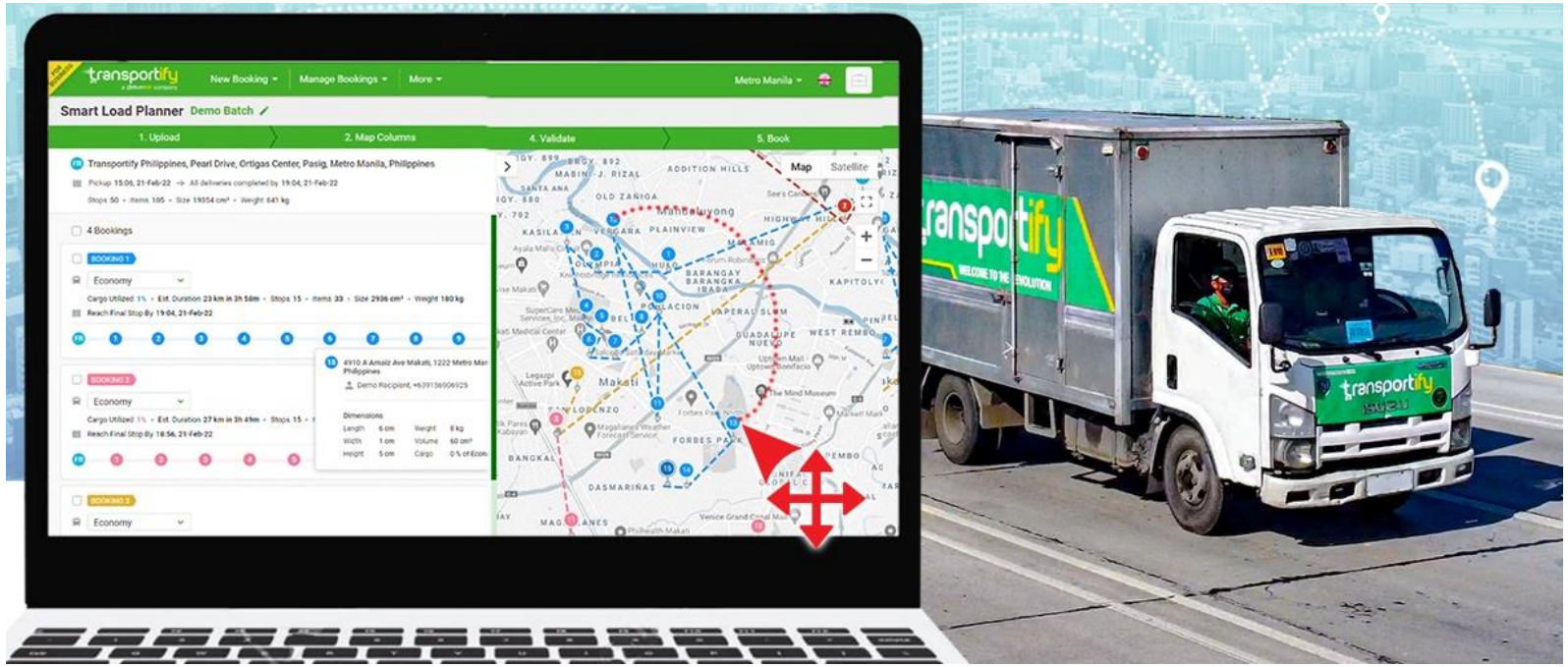


Modern practice: Developing optimization methodologies,
and applying optimization to improve various systems!

Industrial Engineering

Definition

Industrial engineering is an engineering profession that focuses on optimizing products, services, processes, and operations across various systems.



Route planning in logistics

Industrial Engineering

Definition

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Inventory management

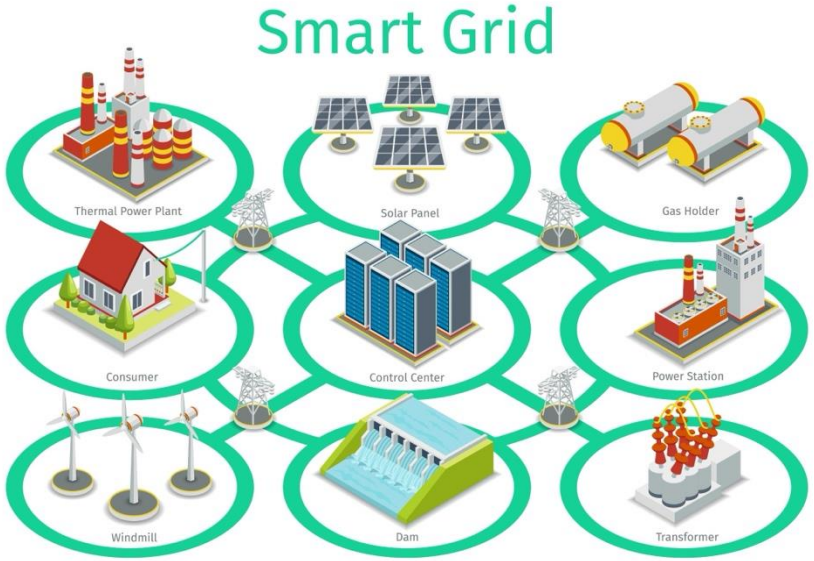
~~Industrial Engineering~~ System Engineering

Definition

Industrial engineering is an engineering profession that focuses on optimizing products, services, processes, and operations across various systems.



Traffic management



Energy management

Optimal Production

How factory determines their production levels?









Optimal Production

Background

Consider a company that produces *n* different goods. The company wants to determine the optimal production quantities of each good to maximize its profit. What factors does the company need to consider?



Resources



Raw Material



Labor



Machine

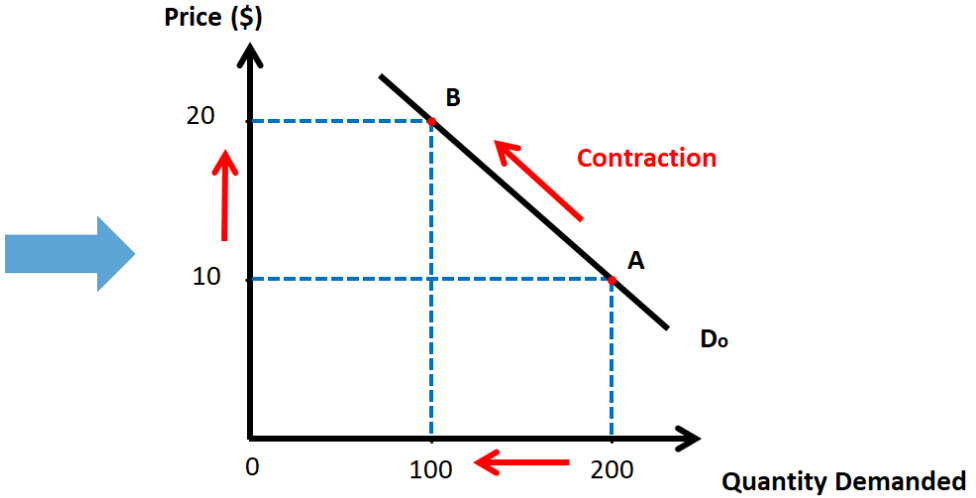
Optimal Production

Background

Consider a company that produces n different goods. The company wants to determine the optimal production quantities of each good to maximize its profit. Suppose that each good requires a certain amount of resources to produce. Let a_{ij} be the amount of resource j required to produce one unit of good i, and let r_j represent the total available amount of resource j.







Optimal Production

Question

Consider a company that produces n different goods. The company wants to determine the optimal production quantities of each good to maximize its profit.

- Suppose that each good requires a certain amount of resources to produce. Let a_{ij} be the amount of resource j required to produce one unit of good i, and let r_j represent the total available amount of resource j ($j = 1, \dots, m$).

- Suppose that for each good, the price p_i and the demand d_i are associated with the following relationship:

$$p_i = \frac{p_i^0}{1 + \beta_i \cdot d_i}$$

What is the optimal production x_i for every good i?

Optimal Production

Question

Consider a company that produces n different goods. The company wants to determine the optimal production quantities of each good to maximize its profit.

- a_{ij}: the amount of resource j required to produce one unit of good i
- r_j: represent the total available amount of resource j
- c_i: the cost of producing a unit of good i
- p_i: the price of good i, where $p_i = p_i^0/(1 + \beta_i \cdot d_i)$ (d_i is the demand of good i)
- x_i : the units of good i to be produced

$$\text{maximize} \quad \sum_{i=1}^{n} \frac{p_i^0}{1+\beta_i \cdot x_i} \cdot x_i - c_i \cdot x_i$$

$$\text{subject to} \quad \sum_{i=1}^{n} a_{ij} \cdot x_i \le r_j, \quad j = 1, \dots, m$$

$$x_i \ge 0, \qquad i = 1, \dots, n$$

⇒ A convex program

Portfolio Optimization

If you have 1 million dollar, how would you invest in various assets to maximize the return?

Asset: anything that can be used to produce positive economic value.



Stock



Bond

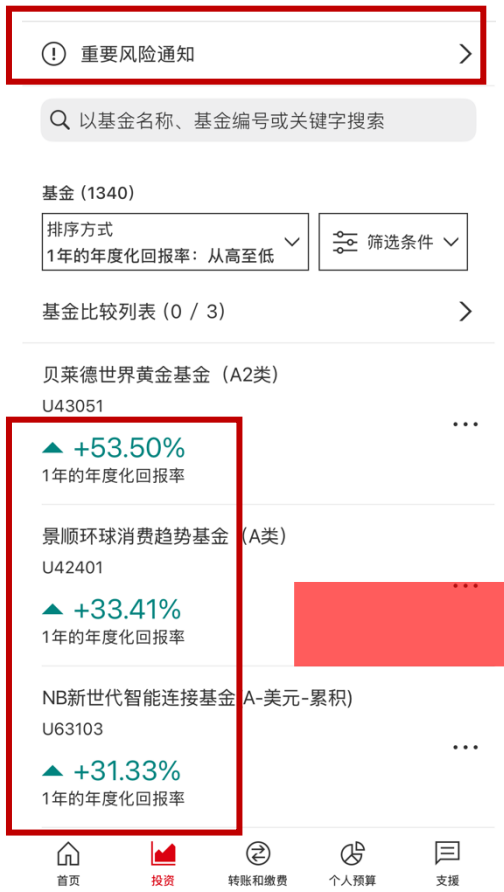
…



Real Estate

Portfolio Optimization

The return rate of any asset can be stochastic.





The numbers can be extremely risky!

Portfolio Optimization

The return rate of any asset can be stochastic.

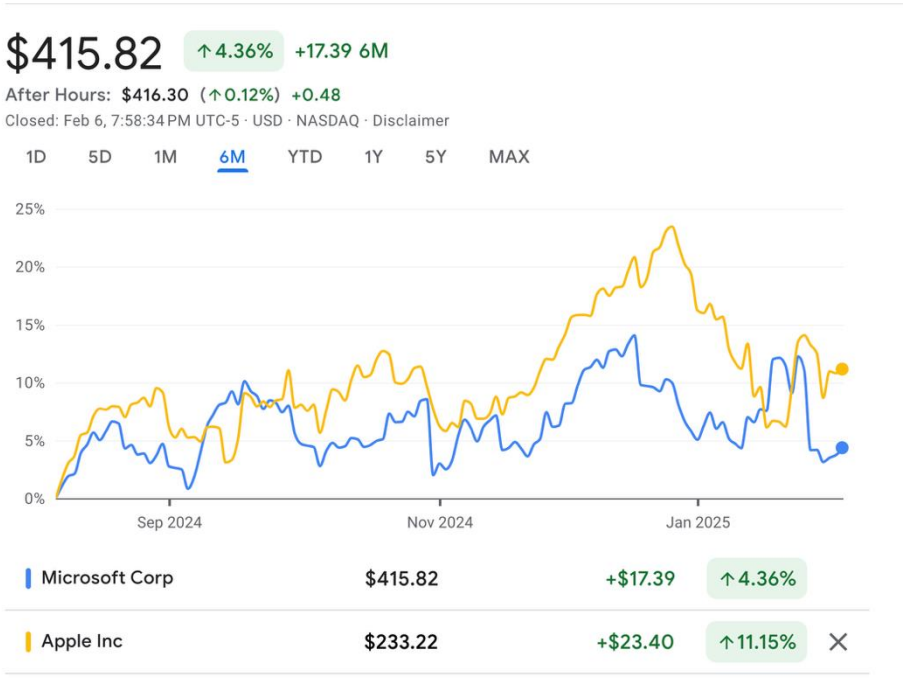


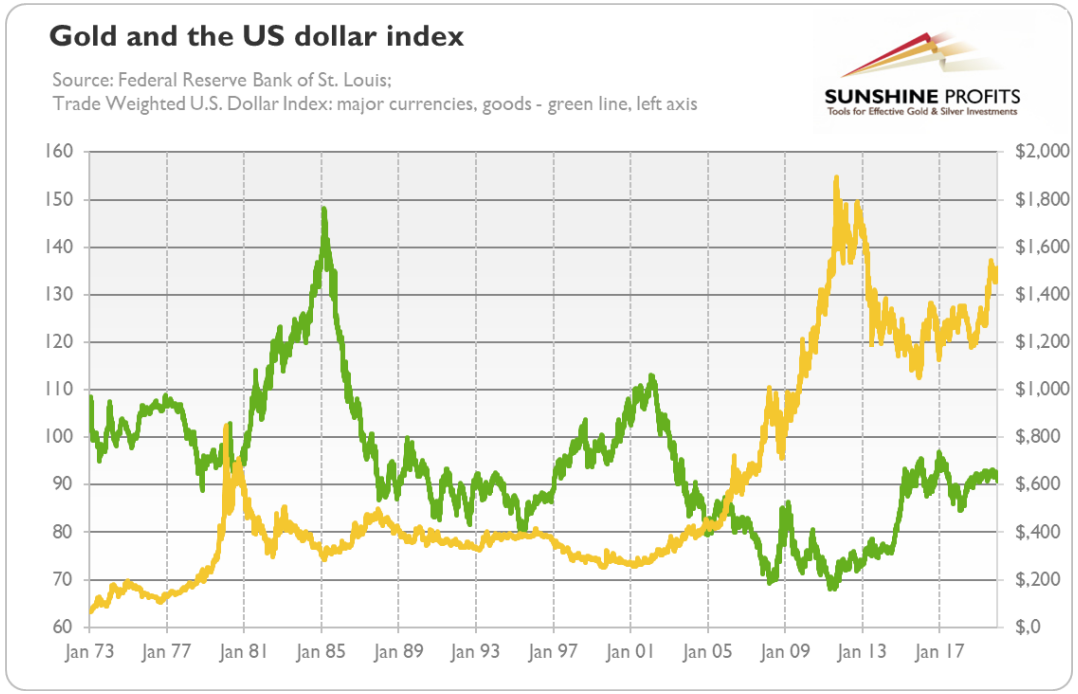
This number is almost risk-free, but…

Search what happened to Long-Term Capital Management
(a hedge fund operated by two Nobel laurates)!

Portfolio Optimization

The risks of two assets can be correlated to each other
because they are affected by the same set of factors





Positive: Microsoft and Apple

Negative: Gold and US Dollar

Portfolio Optimization

Question

Given n assets whose return rates are R_i $(i = 1, \dots, n)$, where $\mathbb{E}(\mathrm{R}_i) = r_i$ $(i = 1, \dots, n)$ and $\mathrm{cov}(\mathrm{R}_i, \mathrm{R}_j) = \sigma_{ij}$ $(i, j = 1, \dots, n)$, let x_i be the the fraction of the total portfolio value invested in Asset i. What x_i $(i = 1, \dots, n)$ make a good portfolio?

The return rate of the portfolio: $\bar{\mathrm{R}} = \sum_{i=1}^{n} x_i \cdot \mathrm{R}_i$.

$\mathbb{E}(\bar{\mathrm{R}})$ as high as possible

$\mathrm{var}(\bar{\mathrm{R}})$ as low as possible

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \mathrm{R} = \begin{bmatrix} \mathrm{R}_1 \\ \vdots \\ \mathrm{R}_n \end{bmatrix} \quad r = \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix}$$

$$\mathbb{E}(\bar{\mathrm{R}}) = \mathbb{E}(x^{\mathrm{T}}\mathrm{R}) = x^{\mathrm{T}} r$$

$$\mathrm{var}(\bar{\mathrm{R}}) = \mathrm{var}(x^{\mathrm{T}}\mathrm{R}) = x^{T}\Sigma x$$

Portfolio Optimization

Question

Given n assets whose return rate are R_i $(i = 1, \dots, n)$, where $\mathbb{E}(\mathrm{R}_i) = r_i$ $(i = 1, \dots, n)$ and $\mathrm{cov}(\mathrm{R}_i, \mathrm{R}_j) = \sigma_{ij}$ $(i, j = 1, \dots, n)$, let x_i be the the fraction of the total portfolio value invested in Asset i. How to optimize x_i $(i = 1, \dots, n)$?

$$\mathbb{E}(\bar{\mathrm{R}}) = \mathbb{E}(x^{\mathrm{T}}\mathrm{R}) = x^{\mathrm{T}}r$$ 

$$\mathrm{var}(\bar{\mathrm{R}}) = \mathrm{var}(x^{\mathrm{T}}\mathrm{R}) = x^{T}\Sigma x$$ 

$$\begin{aligned} \text{minimize} \quad & -x^{\mathrm{T}}r + \gamma \cdot x^{T}\Sigma x \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1 \\ & x_i \geq 0, \qquad i = 1, \dots, m \end{aligned}$$

γ: a positive constant that balances expected return and risk

This is not a common practice!

Portfolio Optimization

Question

Given n assets whose return rate are R_i $(i = 1, \dots, n)$, where $\mathbb{E}(\mathrm{R}_i) = r_i$ $(i = 1, \dots, n)$ and $\mathrm{cov}(\mathrm{R}_i, \mathrm{R}_j) = \sigma_{ij}$ $(i, j = 1, \dots, n)$, let x_i be the the fraction of the total portfolio value invested in Asset i. How to optimize x_i $(i = 1, \dots, n)$?

$$\mathbb{E}(\bar{\mathrm{R}}) = \mathbb{E}(x^{\mathrm{T}}\mathrm{R}) = x^{\mathrm{T}}r$$ 

$$\mathrm{var}(\bar{\mathrm{R}}) = \mathrm{var}(x^{\mathrm{T}}\mathrm{R}) = x^{T}\Sigma x$$ 

$$\begin{aligned} \text{minimize} \quad & x^{T}\Sigma x \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1 \\ & x^{\mathrm{T}}r \geq r_{\text{target}} \\ & x_i \geq 0, \qquad i = 1, \dots, m \end{aligned}$$

large companies often have a target return rate for all their investments.

Portfolio Optimization

Question

Given n assets whose return rate are R_i $(i = 1, \dots, n)$, where $\mathbb{E}(\mathrm{R}_i) = r_i$ $(i = 1, \dots, n)$ and $\mathrm{cov}(\mathrm{R}_i, \mathrm{R}_j) = \sigma_{ij}$ $(i, j = 1, \dots, n)$, let x_i be the the fraction of the total portfolio value invested in Asset i. How to optimize x_i $(i = 1, \dots, n)$?

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$$
\begin{aligned}
\text{minimize} \quad & x^{T}\Sigma x \\
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\end{aligned}
$$



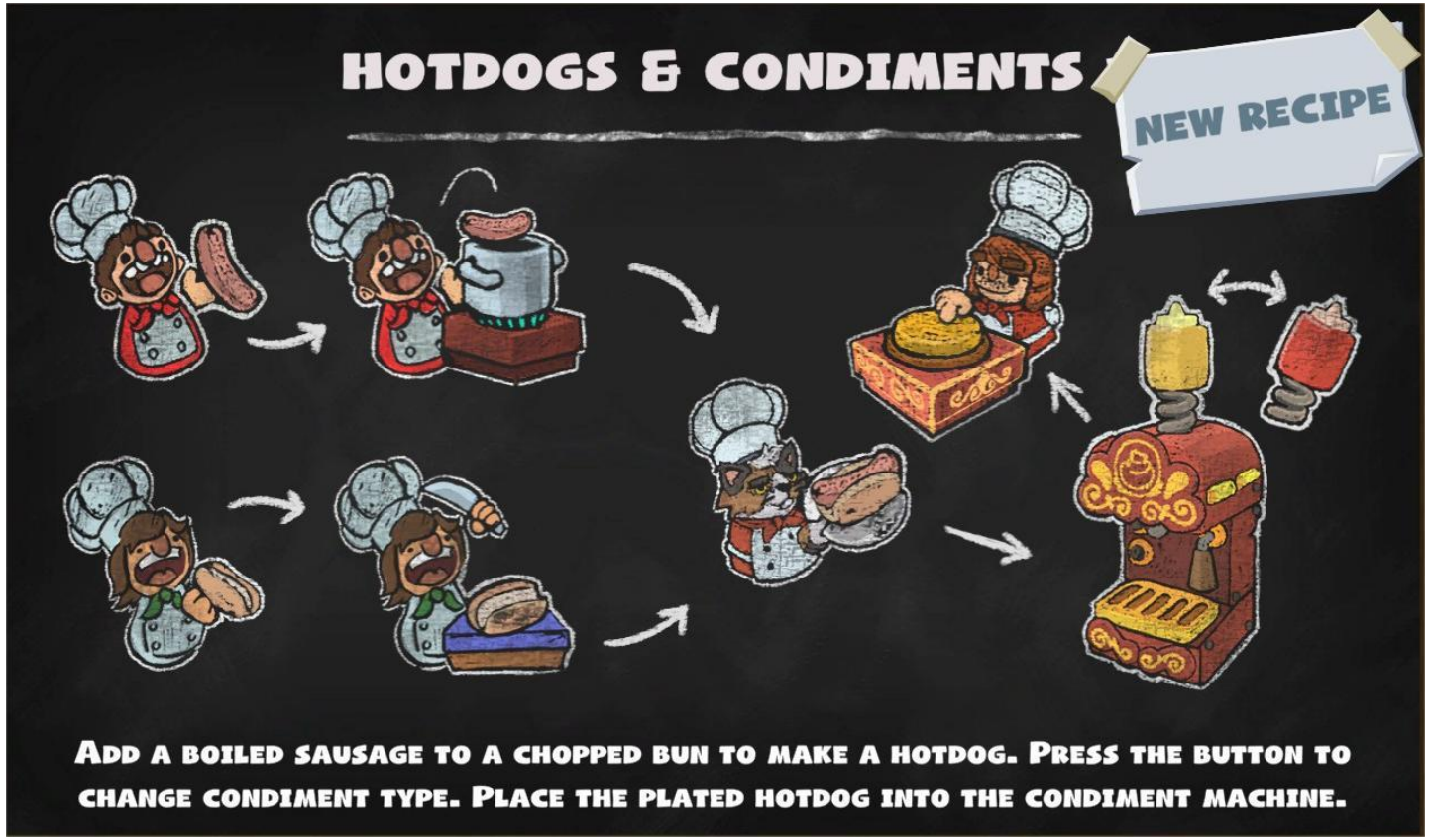
a convex quadratic program

(why convex?)

Project Management

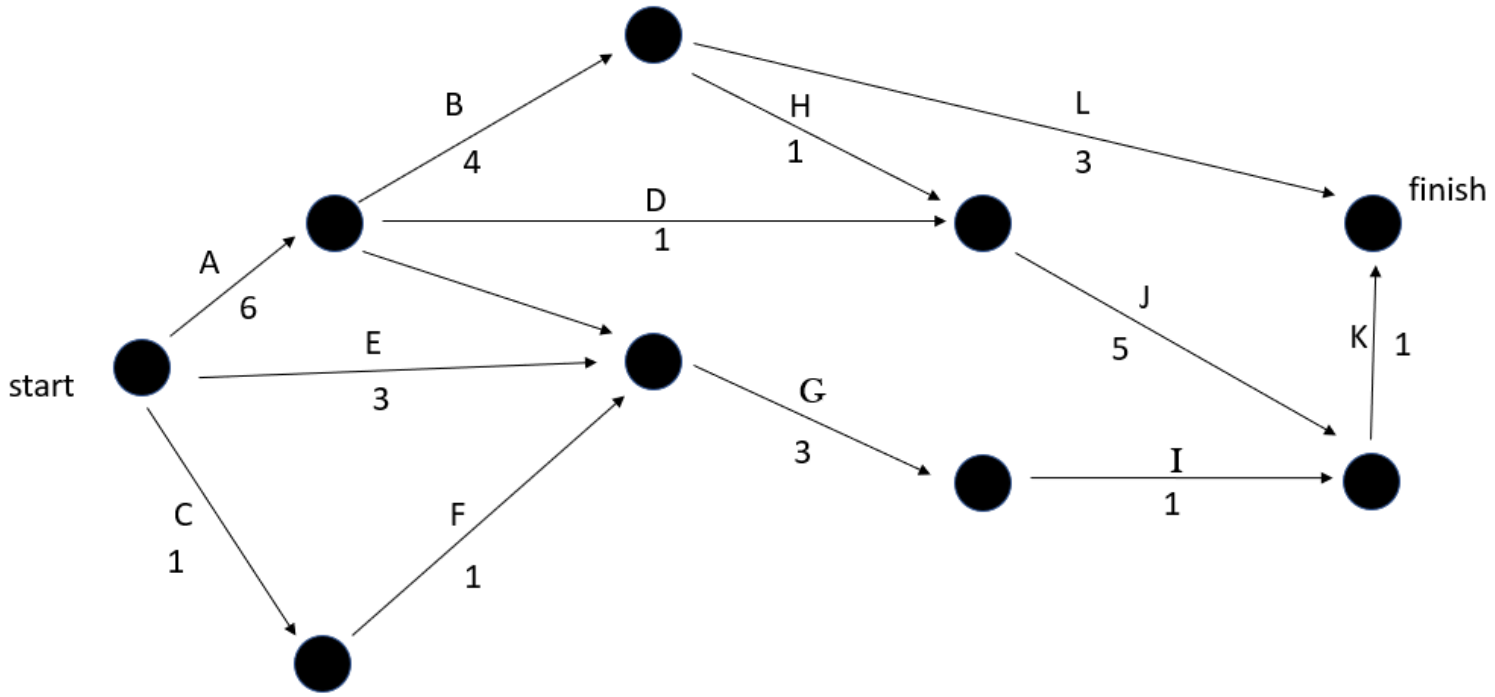
A large project usually involves many subtasks that must be completed in appropriate orders.





Question

What questions does a project manager need to solve before the project is started?



Question

What questions does a project manager need to solve before the project is started?

- **What is the total time required to complete the project?**
- **When does each individual activity need to start and finish (at the latest) to meet this project completion time?**
- **Which are the critical activities where any delays must be avoided to prevent delaying project completion?**
- **For the non-critical activities, how much delay can be tolerated without delaying project completion?**
- **Given the uncertainties in estimating activity durations, what is the probability of completion by a deadline?**
- **If extra money is spent to expedite the project, how would it be used?**
- **……**

Background and Question

A construction firm is building a house for its clients. The following table presents the list of activities that the firm must complete for its current client's project.

Activity	Description	Duration (days)	Predecessor
A	Foundation	8	—
B	Walls and ceilings	10	A
C	Roof	9	B
D	Electrical wiring	8	B, C
E	Windows and doors	5	B
F	Exterior sidings	10	E
G	Utilities	2	C, D, F
H	Landscaping	3	F
I	Painting	10	G, H

What is the earliest time that firm can finish the project?

Background and Question

A construction firm is building a house for its clients. The following table presents the list of activities that the firm must complete for its current client's project.

Activity	Duration (days)	Predecessor	Finished Time
A	8	—	T_A
B	10	A	T_B
C	9	B	T_C
D	8	B, C	T_D
E	5	B	T_E
F	10	E	T_F
G	2	C, D, F	T_G
H	3	F	T_H
I	10	G, H	T_I

$\Rightarrow T_A - 8 \geq 0$

$\Rightarrow T_B - 10 \geq T_A$

$\Rightarrow T_C - 9 \geq T_B$

$\Rightarrow T_D - 8 \geq T_B, T_C$

$\Rightarrow T_E - 5 \geq T_B$

$\Rightarrow T_F - 10 \geq T_E$

$\Rightarrow T_G - 2 \geq T_C, T_D, T_F$

$\Rightarrow T_H - 3 \geq T_F$

$\Rightarrow T_I - 10 \geq T_G, T_H$

What is the earliest time that firm can finish the project? z $\Rightarrow$ $z = \max\{T_A, \ldots, T_I\}$

Background and Question

A construction firm is building a house for its clients. The following table presents the list of activities that the firm must complete for its current client's project.

$$
\begin{aligned}
\text{minimize} \quad & z \\
\text{subject to} \quad & T_A - 8 \geq 0 \\
& T_B - 10 \geq T_A \\
& T_C - 9 \geq T_B \\
& T_D - 8 \geq T_B, T_C \\
& T_E - 5 \geq T_B \\
& T_F - 10 \geq T_E \\
& T_G - 2 \geq T_C, T_D, T_F \\
& T_H - 3 \geq T_F \\
& T_I - 10 \geq T_G, T_H \\
& z \geq T_A, \ldots, T_I
\end{aligned}
$$

$$
\begin{aligned}
\text{minimize} \quad & z \\
\text{subject to} \quad & T_A - 8 \geq 0 \\
& T_B - 10 \geq T_A \\
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& T_H - 3 \geq T_F \\
& T_I - 10 \geq T_G, T_H \\
& z \geq T_A, \ldots, T_I
\end{aligned}
$$

➡ **Linear Program**

The project needs 47 days to be completed.

Background and Question

The firm is then informed that the client hopes the project can be completed in 40 days. Here are the cost of reducing the duration of each activity by a day and the maximum possible reduction.

Activity	Duration	Finished Time
A	8	T_A
B	10	T_B
C	9	T_C
D	8	T_D
E	5	T_E
F	10	T_F
G	2	T_G
H	3	T_H
I	10	T_I

$$T_A - 8 \geq 0$$
$$T_B - 10 \geq T_A$$
$$T_C - 9 \geq T_B$$
$$T_D - 8 \geq T_B, T_C$$
$$T_E - 5 \geq T_B$$
$$T_F - 10 \geq T_E$$
$$T_G - 2 \geq T_C, T_D, T_F$$
$$T_H - 3 \geq T_F$$
$$T_I - 10 \geq T_G, T_H$$

Reduction cost	Maximum
5000	2
9000	2
7000	4
4500	3
1500	1
3500	3
-	0
4500	1
9500	3

What is the minimum additional cost required for the project to be completed in 40 days?

Background and Question

The firm is informed that the client is willing to pay a bonus if the project can be completed in 40 days. Here are the cost of reducing the duration of each activity by a day and the maximum possible reduction.

Activity	Duration	Finished Time
A	$8 - x_A$	T_A
B	$10 - x_B$	T_B
C	$9 - x_C$	T_C
D	$8 - x_D$	T_D
E	$5 - x_E$	T_E
F	$10 - x_F$	T_F
G	$2 - x_G$	T_G
H	$3 - x_H$	T_H
I	$10 - x_I$	T_I

$$T_A - (8 - x_A) \geq 0$$
$$T_B - (10 - x_B) \geq T_A$$
$$T_C - (9 - x_C) \geq T_B$$
$$T_D - (8 - x_D) \geq T_B, T_C$$
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$$T_F - (10 - x_F) \geq T_E$$
$$T_G - (2 - x_G) \geq T_C, T_D, T_F$$
$$T_H - (3 - x_H) \geq T_F$$
$$T_I - (10 - x_I) \geq T_G, T_H$$

$$z = \max\{T_A, \dots, T_I\} \leq 40$$

Reduction	Reduction cost	Maximum
x_A	5000	2
x_B	9000	2
x_C	7000	4
x_D	4500	3
x_E	1500	1
x_F	3500	3
x_G	-	0
x_H	4500	1
x_I	9500	3

Background and Question

The firm is informed that the client is willing to pay a bonus if the project can be completed in 40 days. Here are the cost of reducing the duration of each activity by a day and the maximum possible reduction.

$$\text{minimize} \quad 5000 \cdot x_A + \cdots + 9500 \cdot x_I$$

$$\text{subject to} \quad 0 \le x_A \le 2 \qquad T_A - (8 - x_A) \ge 0$$
$$0 \le x_B \le 2 \qquad T_B - (10 - x_B) \ge T_A$$
$$0 \le x_C \le 4 \qquad T_C - (9 - x_C) \ge T_B$$
$$0 \le x_D \le 3 \qquad T_D - (8 - x_D) \ge T_B, T_C$$
$$0 \le x_E \le 1 \qquad T_E - (5 - x_E) \ge T_B$$
$$0 \le x_F \le 3 \qquad T_F - (10 - x_F) \ge T_E$$
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$$0 \le x_H \le 1 \qquad T_H - (3 - x_H) \ge T_F$$
$$0 \le x_I \le 3 \qquad T_I - (10 - x_I) \ge T_G, T_H$$
$$T_A, \ldots, T_I \le 40 \qquad \sout{z = \max\{T_A, \ldots, T_I\} \le 40}$$

Reduction	Reduction cost	Maximum
x_A	5000	2
x_B	9000	2
x_C	7000	4
x_D	4500	3
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Background and Question

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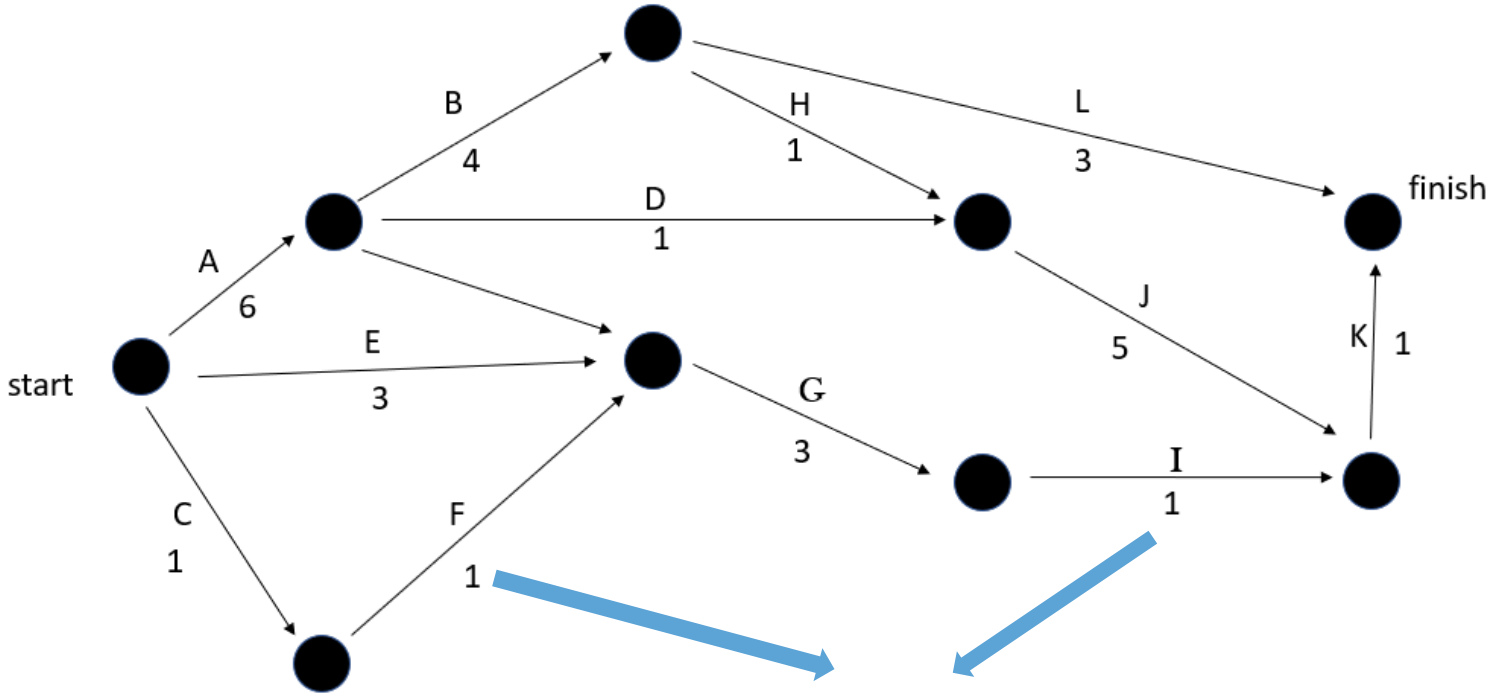
$$\text{minimize} \quad 5000 \cdot x_A + \cdots + 9500 \cdot x_I$$

$$\text{subject to} \quad 0 \le x_A \le 2 \qquad T_A - (8 - x_A) \ge 0$$
$$0 \le x_B \le 2 \qquad T_B - (10 - x_B) \ge T_A$$
$$0 \le x_C \le 4 \qquad T_C - (9 - x_C) \ge T_B$$
$$0 \le x_D \le 3 \qquad T_D - (8 - x_D) \ge T_B, T_C$$
$$0 \le x_E \le 1 \qquad T_E - (5 - x_E) \ge T_B$$
$$0 \le x_F \le 3 \qquad T_F - (10 - x_F) \ge T_E$$
$$x_G = 0 \qquad T_G - (2 - x_G) \ge T_C, T_D, T_F$$
$$0 \le x_H \le 1 \qquad T_H - (3 - x_H) \ge T_F$$
$$0 \le x_I \le 3 \qquad T_I - (10 - x_I) \ge T_G, T_H$$
$$T_A, \ldots, T_I \le 40 \qquad z = \max\{T_A, \ldots, T_I\} \le 40$$

⇒ **Linear Program**

Project Management

Project management can be much more challenge in reality!



The duration of every activity may be uncertain…

Project Management

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Location Decision

Definition

A **location decision** problem involves determining the **optimal placement** of facilities such as stores, warehouses, factories, retail outlets, or service centers to maximize operational efficiency.



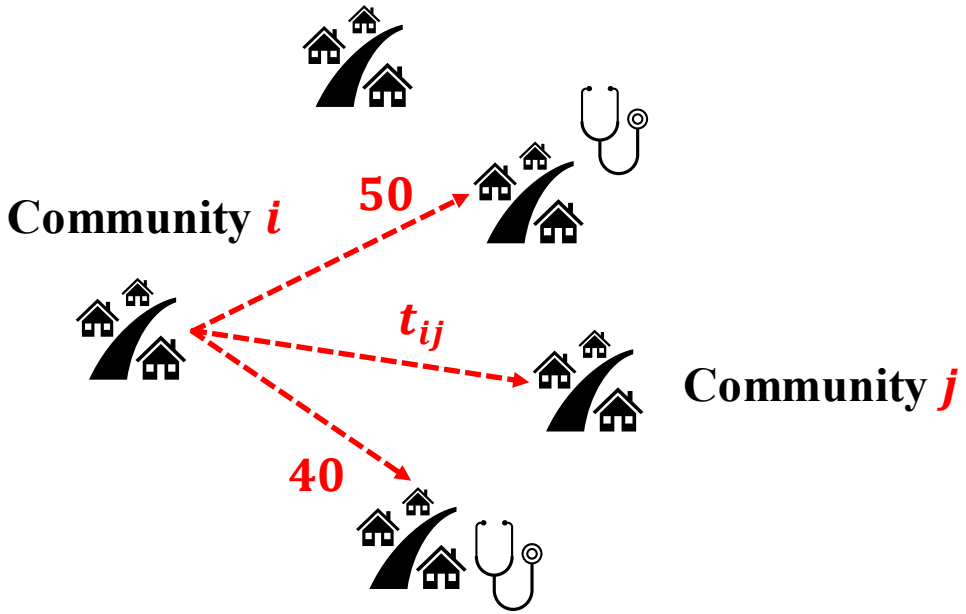




Location Decision

Background

The government is planning to open medical clinics to serve n communities in the rural area. The time for driving from Community i to Community j is t_{ij} minutes. The plan is to locate the clinics in these communities so every community will be within 30 minutes of drive of at least one one clinic.



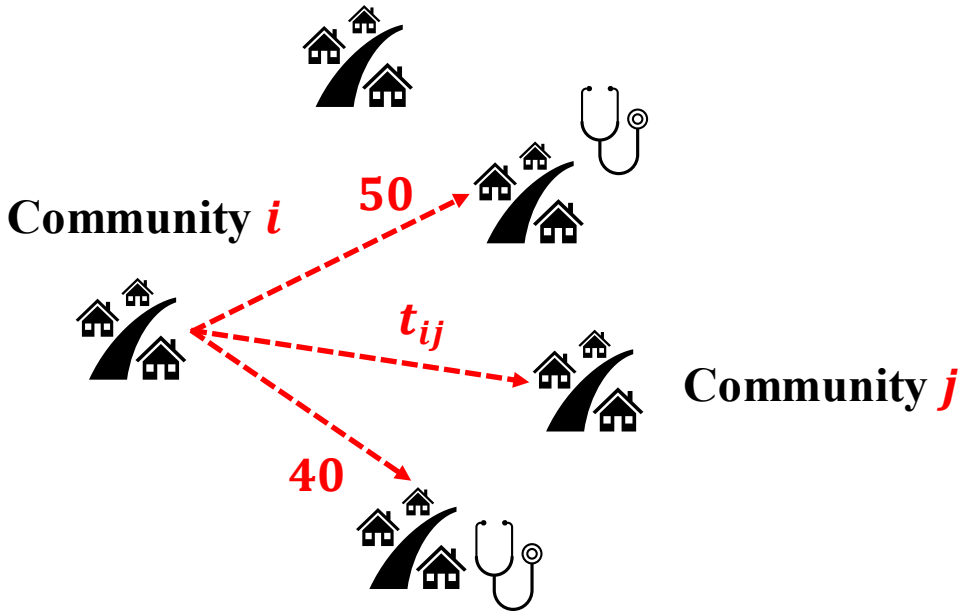
Question

If any community can serve as a potential clinic site, what is the minimum number of medical clinics that covers all communities? And where these clinics must be located?

Location Decision

Question

If any community can serve as a potential clinic site, what is the minimum number of medical clinics that covers all communities? And where these clinics must be located?



$$x_i = \begin{cases} 1 & \text{if a clinic is located in Community } i \\ 0 & \text{otherwise} \end{cases}$$

$$d_{ij} = \begin{cases} 1 & \text{if } t_{ij} \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

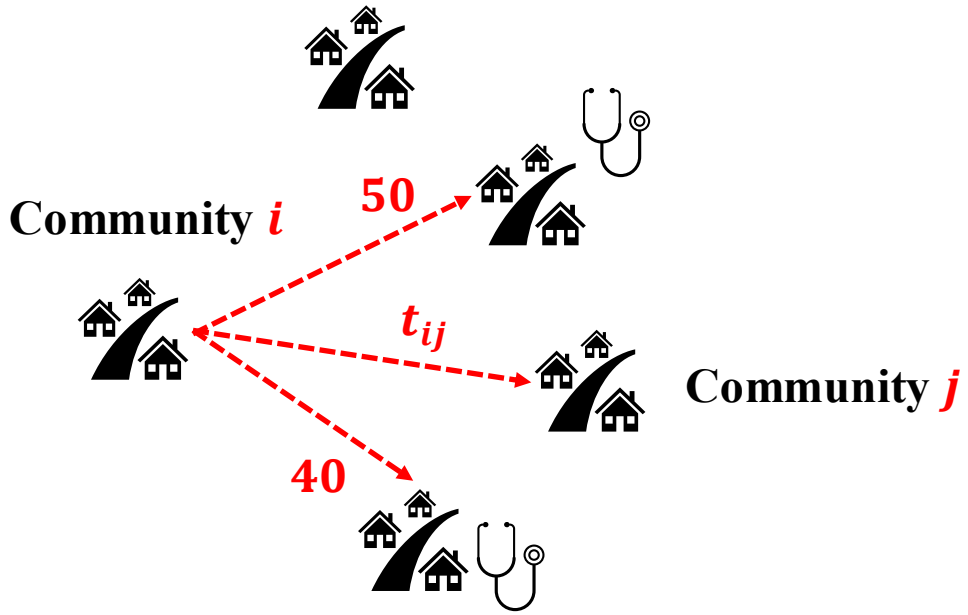
What is $\sum_{j=1}^{n} d_{ij} \cdot x_j$?

The total number of clinics which can be reached from Community i within 30 minutes

Location Decision

Question

If any community can serve as a potential clinic site, what is the minimum number of medical clinics that covers all communities? And where these clinics must be located?



$$x_i = \begin{cases} 1 & \text{if a clinic is located in Community } i \\ 0 & \text{otherwise} \end{cases}$$

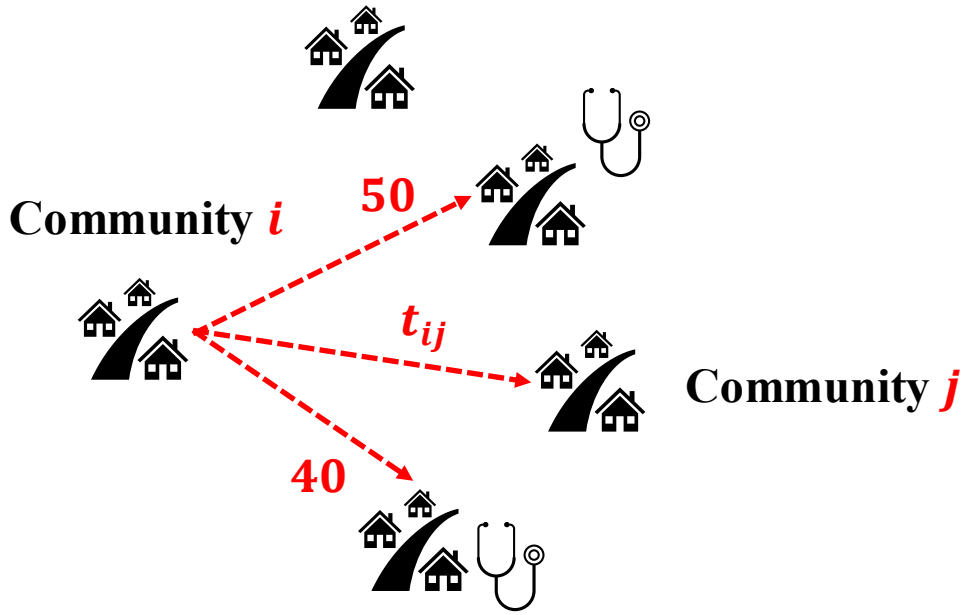
$$d_{ij} = \begin{cases} 1 & \text{if } t_{ij} \le 30 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} x_i \\ \text{subject to} \quad & \sum_{j=1}^{n} d_{ij} \cdot x_j \ge 1, \quad i = 1, \dots, n \\ & x_i \in \{0, 1\}, \qquad i = 1, \dots, n \end{aligned}$$

Location Decision

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If any community can serve as a potential clinic site, what is the minimum number of medical clinics that covers all communities? And where these clinics must be located?



Linear Program

Mixed-Integer Linear Program

$$\text{minimize} \quad \sum_{i=1}^{n} x_i$$

$$\text{subject to} \quad \sum_{j=1}^{n} d_{ij} \cdot x_j \geq 1, \quad i = 1, \ldots, n$$

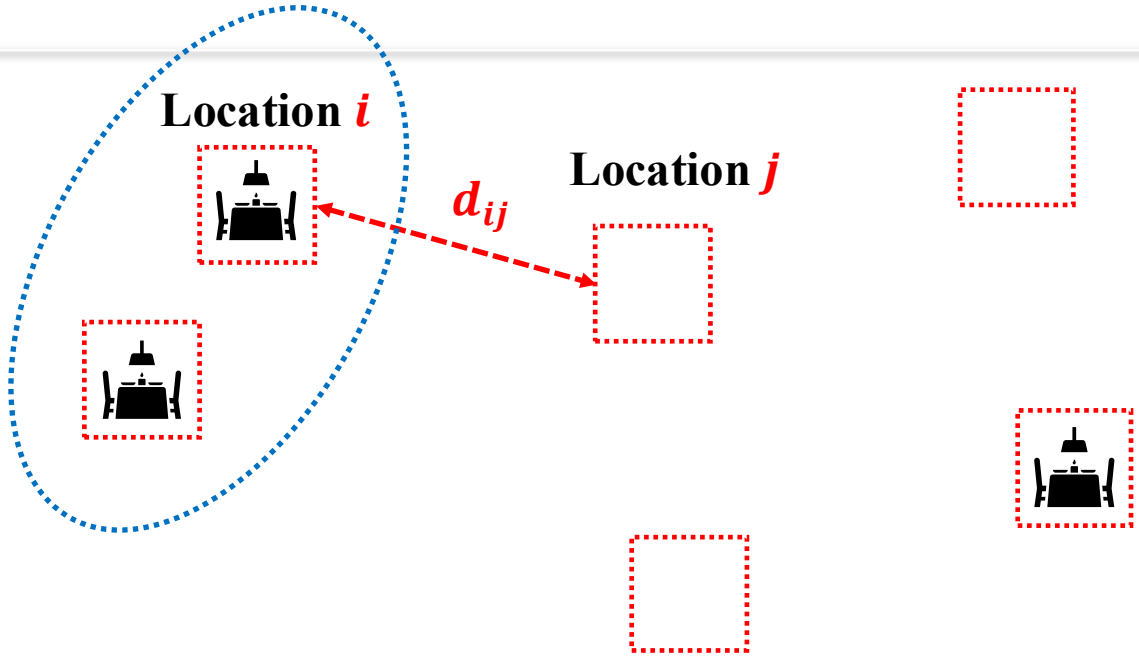
$$x_i \in \{0, 1\}, \quad i = 1, \ldots, n$$

Location Decision

Background

A dining company is planning to open n restaurants in a city and it has identifies m $(m \geq n)$ potential locations. The distance between Locations i and j is denoted as d_{ij}. In order to prevent cannibalization of sales (the restaurants competing with each other), the firm would like to have their n restaurants be located as far from each other as possible.

The distance between the two closest selected locations as large as possible



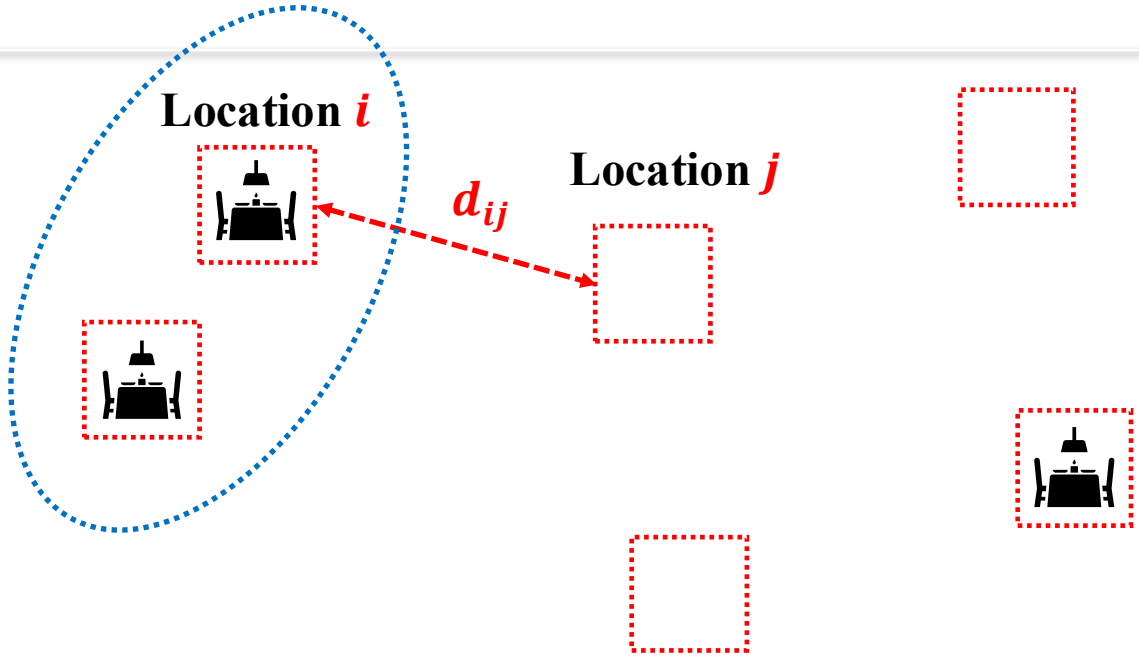
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Question

Where should the n restaurants be located?



Location Decision

$$x_i = \begin{cases} 1 & \text{if a restaurant is opened in Location } i \\ 0 & \text{otherwise} \end{cases}$$

d_{ij}: the distance between Locations i and j

w: the distance between two closest selected locations.

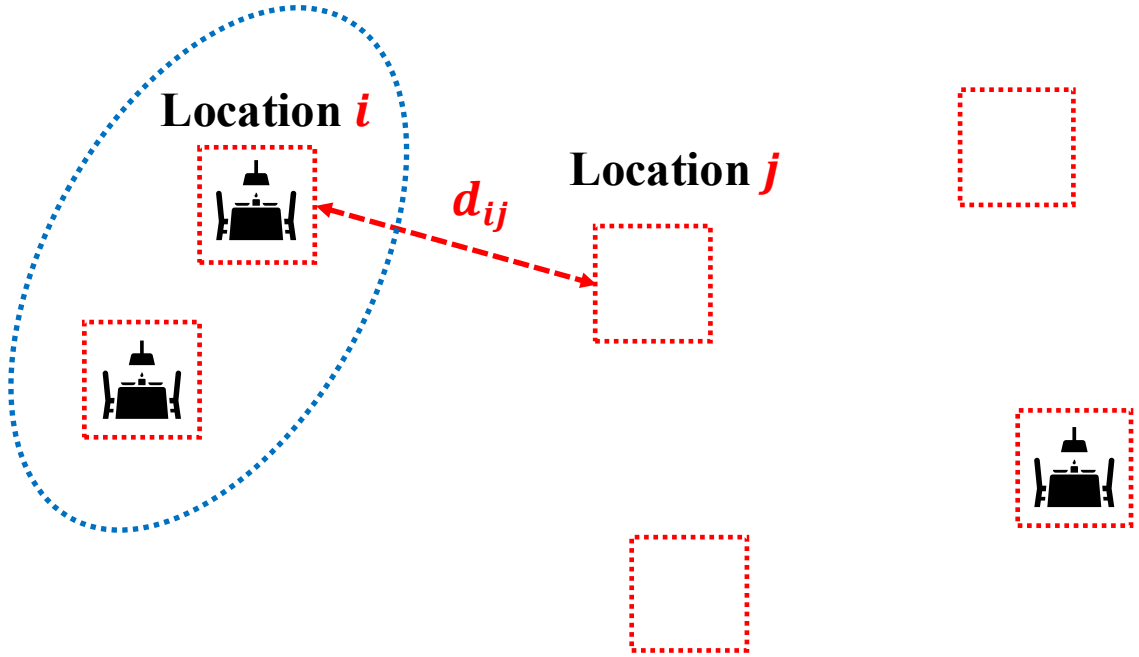
$$w = \min\{d_{ij}: x_i = 1, x_j = 1, \; i, j = 1, \ldots, , m, \; i \neq j\}$$

maximize w

subject to $w = \min\{d_{ij}: x_i = 1, x_j = 1, \; i, j = 1, \ldots, , m, \; i \neq j\}$

$$\sum_{i=1}^{n} x_i = n$$

$$x_i \in \{0, 1\}, \qquad i = 1, \ldots, m$$



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Question

Can you reformulate the problem as a mixed-integer linear program (MILP)?

Location Decision

$$x_i = \begin{cases} 1 & \text{if a restaurant is opened in Location } i \\ 0 & \text{otherwise} \end{cases}$$

w: the distance between two closest selected locations.

d_{ij}: the distance between Locations i and j

➡ M: a constant $> d_{ij}$, $\forall i, j = 1, \dots, , m,\ i \neq j$

➡ $e_{ij} = d_{ij} + M \cdot (2 - x_i - x_j)$

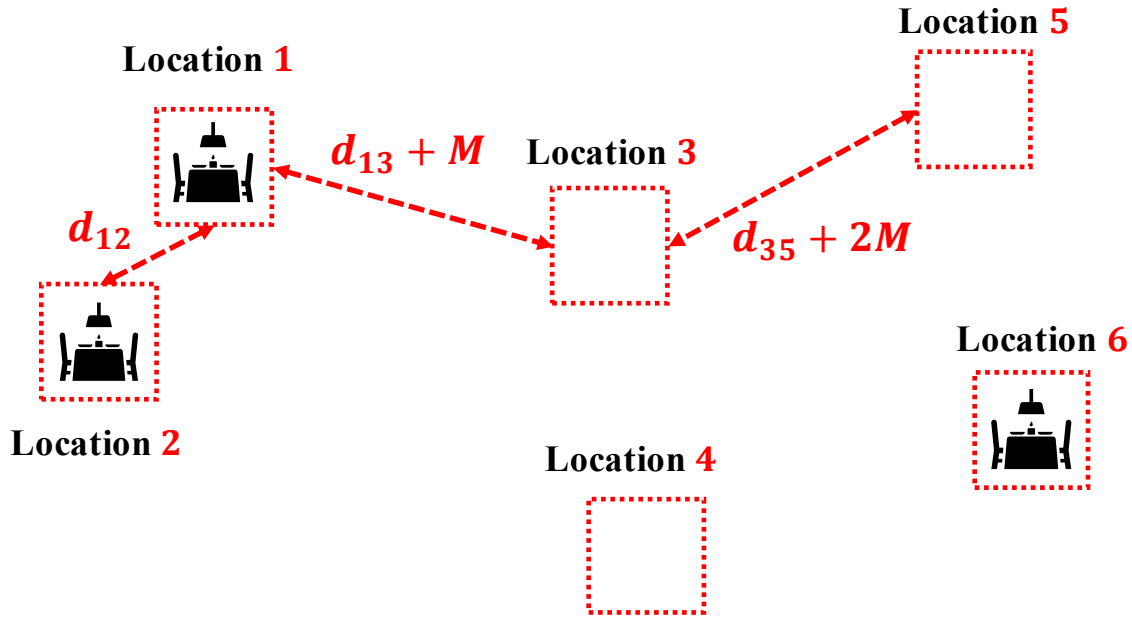
maximize w

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~~e_{ij}~~

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$\sum_{i=1}^{n} x_i = n$

$x_i \in \{0, 1\}, \quad i = 1, \dots, m$

➡

maximize w

subject to $w = \min \{d_{ij} + M \cdot (2 - x_i - x_j): \ i, j = 1, \dots, , m, \ i \neq j\}$

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M: a constant $> d_{ij}$, $\forall i, j = 1, \dots, , m,\ i \neq j$

w: the distance between two closest selected locations.

maximize w

subject to $w \leq d_{ij} + M \cdot (2 - x_i - x_j)$,
$i, j = 1, \dots, , m,\ i \neq j$

$\sum_{i=1}^{n} x_i = n$

$x_i \in \{0, 1\}, \qquad i = 1, \dots, m$

MILP

maximize w

subject to $w = \min \{d_{ij} + M \cdot (2 - x_i - x_j):$
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Location Decision

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The distance between the two closest selected locations as large as possible

Question

Is this indeed a good practice?

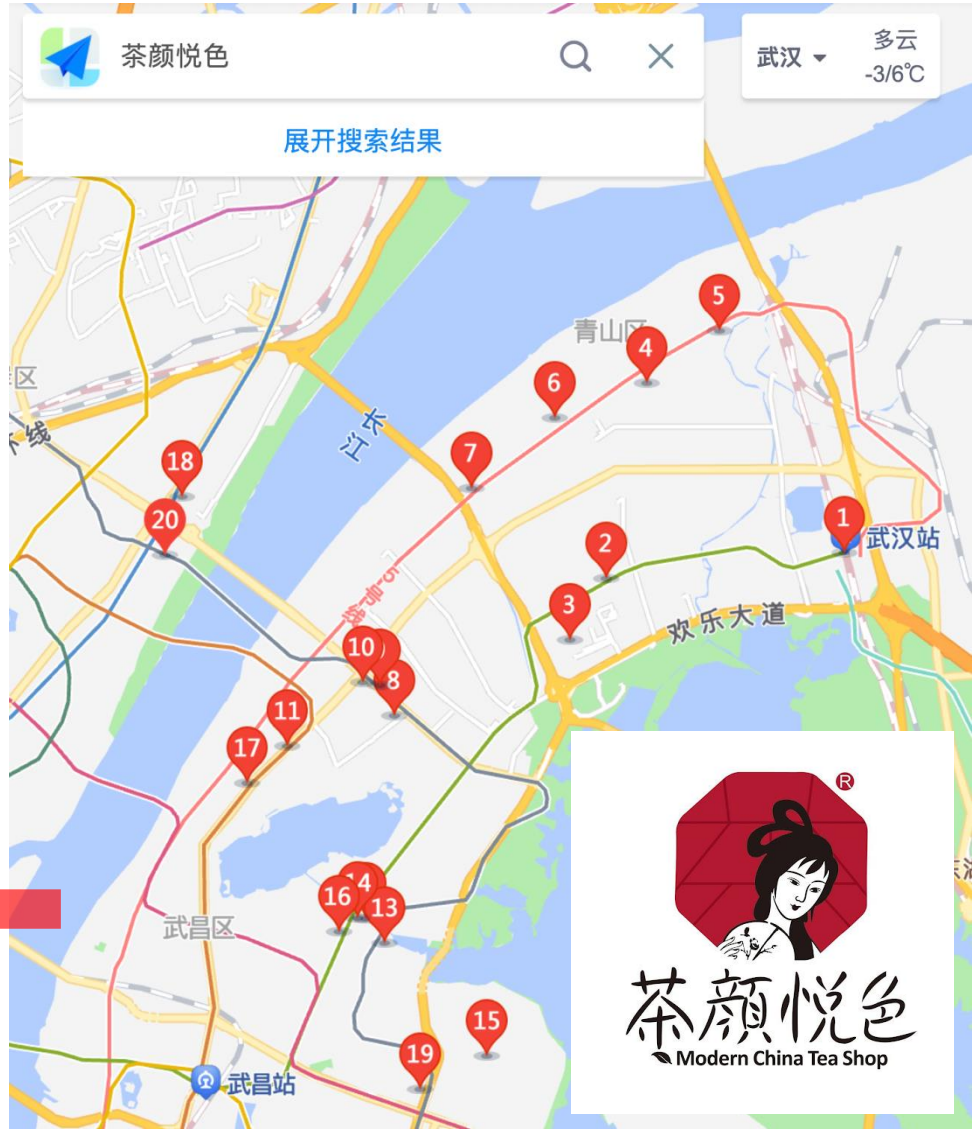
Location Decision

The distance between the two closest selected locations as large as possible

Question

Is this indeed a good practice?

Clustering may also trigger impulse buying!

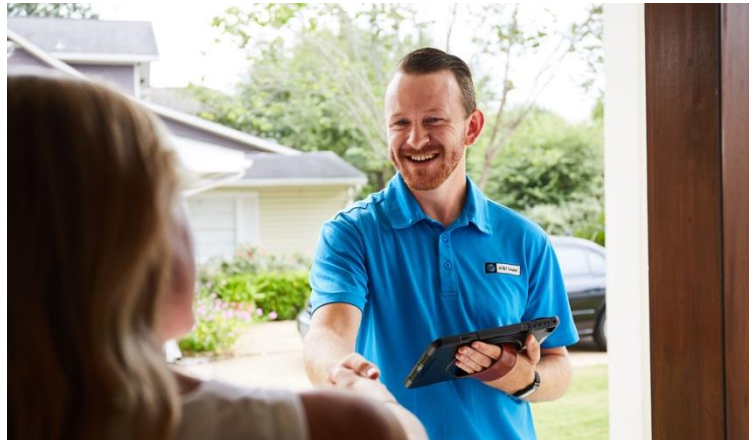


Traveling Salesman

Suppose there are a list of points of interests you need to be visited, how would you plan your routes?





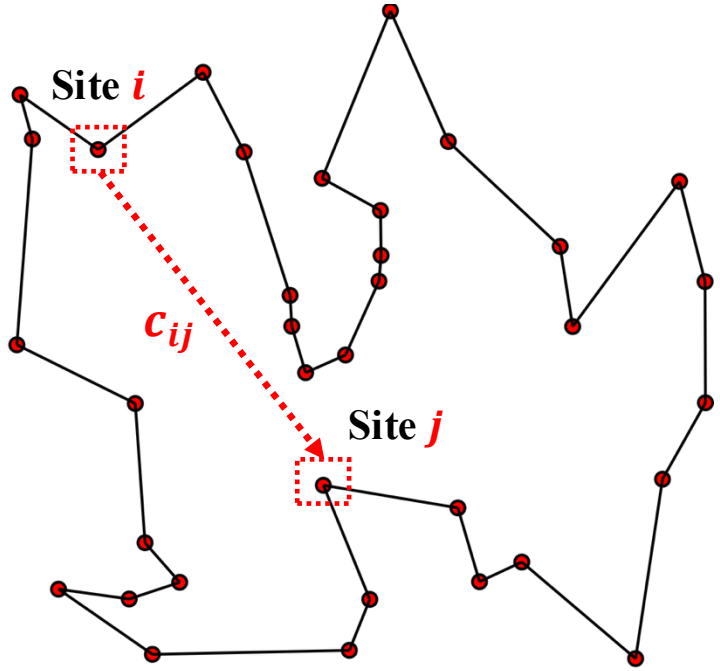




Traveling Salesman

Question

Given a list of n sites, supposing that the distance from traveling from Site i to Site j is c_{ij}, what is the shortest possible path that visits each city exactly once and returns to the origin city?



Decision variable:

$$x_{ij} = \begin{cases} 1 & \text{if the path goes from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

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Given a list of n sites, supposing that the distance of traveling from Site i to Site j is c_{ij}, what is the shortest possible path that visits each city exactly once and returns to the origin city?

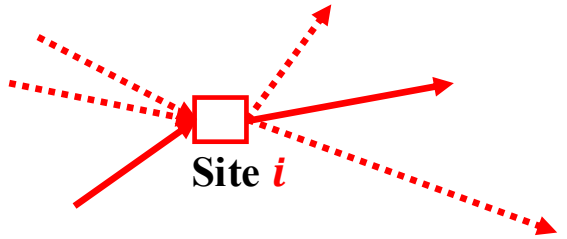
$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j \neq i, j=1}^{n} c_{ij} \cdot x_{ij}$$

$$\text{subject to} \quad \begin{cases} \sum_{j \neq i, j=1}^{n} x_{ij} = 1, & i = 1, \dots, n \\ \sum_{j \neq i, j=1}^{n} x_{ji} = 1, & i = 1, \dots, n \end{cases}$$

$$x_{ij} \in \{0, 1\}, \qquad i \neq j, i, j = 1, \dots, n$$

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$$x_{ij} = \begin{cases} 1 & \text{if the path goes from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$



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minimize $\sum_{i=1}^{n} \sum_{j \neq i, j=1}^{n} c_{ij} \cdot x_{ij}$

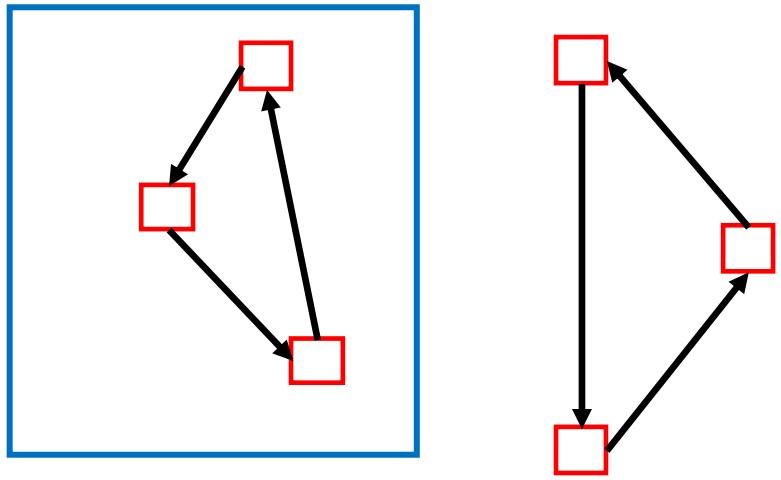
subject to

$$\sum_{j \neq i, j=1}^{n} x_{ij} = 1, \quad i = 1, \dots, n$$

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A subtour

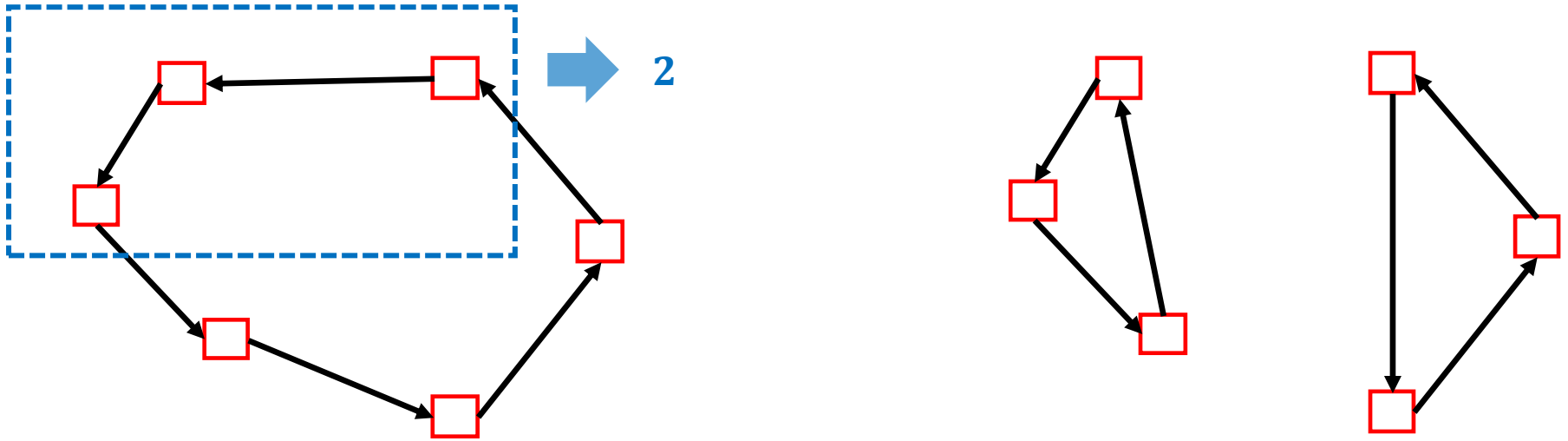


Does this path satisfy all these constraints?

Traveling Salesman

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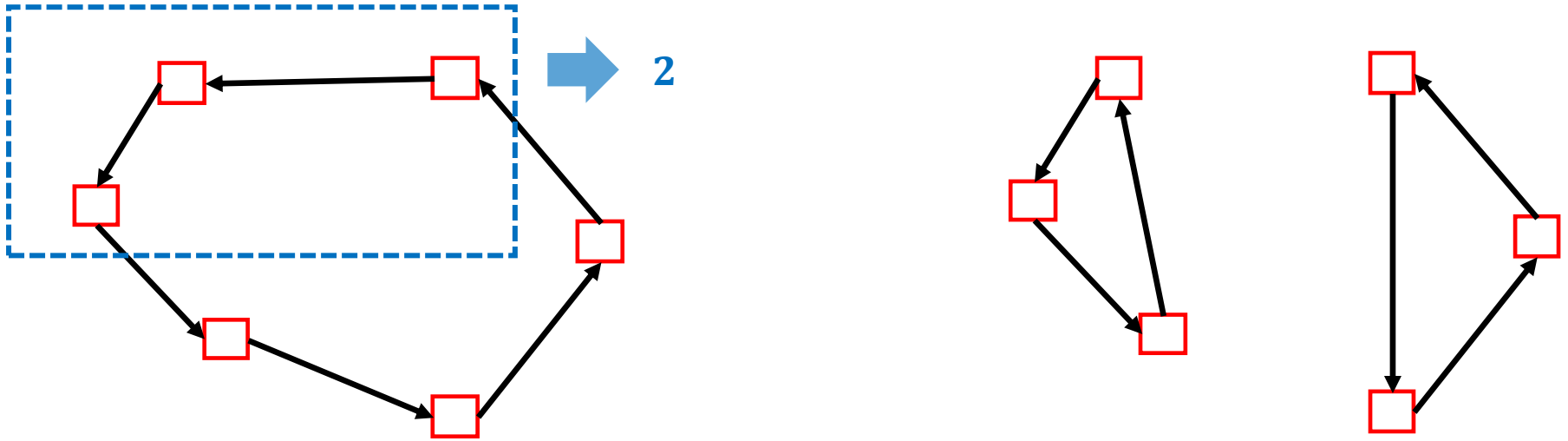


For all $Q \subset \{1, \dots, n\}$ with $|Q| = 3$, please compute $\sum_{i \in Q} \sum_{j \neq i, j \in Q} x_{ij}$

Traveling Salesman

Question

Given a list of n sites, supposing that the distance of traveling from Site i to Site j is c_{ij}, what is the shortest possible path that visits each city exactly once and returns to the origin city?



Can we say as long as for all $Q \subset \{1, \dots, n\}$ with $|Q| = 3$, $\sum_{i \in Q} \sum_{j \neq i, j \in Q} x_{ij} < 3$, there will be no subtour whose length is 3?

Traveling Salesman

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Given a list of n sites, supposing that the distance of traveling from Site i to Site j is c_{ij}, what is the shortest possible path that visits each city exactly once and returns to the origin city?

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j \neq i, j=1}^{n} c_{ij} \cdot x_{ij}$$

$$\text{subject to} \quad \sum_{j \neq i, j=1}^{n} x_{ij} = 1, \quad i = 1, \dots, n$$

$$\sum_{j \neq i, j=1}^{n} x_{ji} = 1, \quad i = 1, \dots, n$$

$$\sum_{i \in Q} \sum_{j \neq i, j \in Q} x_{ij} \leq |Q| - 1, \quad \forall Q \subset \{1, \dots, n\}, \quad 1 < |Q| < n$$

$$x_{ij} \in \{0, 1\}, \qquad i \neq j, i, j = 1, \dots, n$$

→ MILP